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Right Congruences on Completely Inverse AG**-Groupoids

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Dedicated to the memory of Dr Nebojša Stevanović, Faculty of Civil Engineering, University of Niš, in recognition of his contribution to the development of the theory.

Abstract. We revisit right congruences on completely inverse AG**-groupoids. The starting point is a short observation: every right congruence on such a groupoid is automatically a two-sided congruence. This collapses the one-sided theory to the ordinary congruence theory and permits the use of the published kernel–trace description of congruences. We then reconstruct, correct, and streamline a right-congruence-pair framework originating in the author's 1997 doctoral dissertation and not subsequently published in journal form. For a pseudo-right congruence pair (K, τ) we construct a right congruence $\rho_{(K,\tau)}$, determine its trace and kernel, and obtain an exact representation for right congruence pairs. Finally, we compare this framework with the standard congruence-pair theory for completely inverse AG**-groupoids: right congruence pairs and ordinary congruence pairs encode exactly the same kernel–trace data, and the apparently more complicated one-sided relation reduces to the standard kernel–trace criterion. As a consequence, the lattice of right congruences coincides with the congruence lattice.

1. Introduction

Abel–Grassmann groupoids (also called *AG-groupoids*, *LA-semigroups* or *left-invertive groupoids* in parts of the literature) are groupoids satisfying the left-invertive identity

$$(xy)z = (zy)x.$$

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An *AG**-groupoid* additionally satisfies

$$x(yz) = y(xz).$$

Congruences on AG**-groupoids and inverse AG**-groupoids were studied in several early papers, including Protić and Božinović [9], the natural-order approach of Božinović–Protić–Stevanović [2], the kernel-normal-system approach [3], and Protić's description via the natural partial order [8]. Later, Dudek and Gigoń developed the structure and congruence theory of completely inverse AG**-groupoids, including a kernel–trace characterization and an abstract congruence-pair theorem [4, 5]. Subsequent work continued the study of congruence pairs, congruence structure, and related structural questions for completely inverse or closely related AG**-groupoids [6, 7, 10].

The present paper concerns a one-sided question: what additional freedom is obtained if compatibility is required only on the right? A *right congruence* is an equivalence relation ρ such that

$$a \,\rho\, b \quad \Longrightarrow \quad ac \,\rho\, bc \qquad (a, b, c \in S).$$

For general nonassociative systems this is genuinely weaker than two-sided compatibility. In a completely inverse AG**-groupoid, however, the distinction disappears. The elementary identity used in Section 3 converts right compatibility into left compatibility.

This observation has two consequences. First, every published theorem about ordinary congruences on completely inverse AG**-groupoids applies immediately to right congruences. Second, it clarifies an unpublished construction from Chapter 6 of the author's doctoral dissertation [1], where pseudo-right and right congruence pairs were introduced. The original dissertation treatment contained a longer representation argument and, in its final converse theorem, additional hypotheses that are unnecessary once the collapse theorem is isolated at the outset.

The contributions of this paper are therefore deliberately formulated relative to the existing bilateral theory. We do not claim a new kernel–trace theory for ordinary congruences. Rather, we: (i) prove that right congruences and congruences coincide on completely inverse AG**-groupoids; (ii) give a corrected self-contained reconstruction of the dissertation's right-pair representation; (iii) remove the additional assumptions from the original converse theorem; and (iv) prove that the right-pair formalism and the standard published congruence-pair formalism encode the same kernel–trace data.

2. Preliminaries and the published bilateral theory

Throughout, S denotes a completely inverse AG**-groupoid. Thus

$$(xy)z = (zy)x, \qquad x(yz) = y(xz),$$

and every $a \in S$ has a unique inverse a^{-1} satisfying

$$a = (aa^{-1})a, \qquad a^{-1} = (a^{-1}a)a^{-1}, \qquad aa^{-1} = a^{-1}a \in E(S).$$

As usual,

$$E(S) = \{e \in S : e^2 = e\}$$

denotes the set of idempotents of S. We use the medial law and the product-inverse rule

$$(xy)(zw) = (xz)(yw), \qquad (xy)^{-1} = x^{-1}y^{-1},$$

as well as the fact that $E(S)$ is a semilattice. In particular, multiplication of idempotents is associative and commutative. We also use

$$e(xy) = (ex)y \quad (e \in E(S),\ x, y \in S).$$

These are standard facts for completely inverse AG^{**}-groupoids [4, 5]. For the proof below we recall the stabdard structur theorem; for each $e \in E(S)$, the set

$$G_e = \{e \in S : xx^{-1} = e\}$$

is an AG-group with left identity e; in particularm its left translations are injective[5, theorem3.4].

The natural partial order is defined by

$$a \leqslant b \iff a = eb \text{ for some } e \in E(S).$$

The following form will be used repeatedly.

Lemma 2.1. *For $a, b \in S$ the following conditions are equivalent:*

(i) $a \leqslant b$;
(ii) $aa^{-1} = a^{-1}b$;
(iii) $a = (ab^{-1})a$;
(iv) $a = (aa^{-1})b$.

Proof. If $a = eb$ with $e \in E(S)$, then

$$aa^{-1} = (eb)(eb^{-1}) = e(bb^{-1}) = (bb^{-1})e = (eb^{-1})b = a^{-1}b.$$

Thus $(i) \Rightarrow (ii)$. From (ii),

$$a^{-1} = (a^{-1}a)a^{-1} = (aa^{-1})a^{-1} = (a^{-1}b)a^{-1},$$

and inversion gives (iii).

To prove $(iii) \Rightarrow (iv)$, put $p = aa^{-1}$ and $q = bb^{-1}$. Inverting (iii) and using the medal law yields

$$p = ((ab^{-1})a)((a^{-1}b)a^{-1}) = ((ab^{-1})(a^{-1}b))(aa^{-1}) = (pq)p = pq.$$

Let $d = pb$. Then $dd^{-1} = pq = p$, so $a, d \in G_p$, and $d^{-1} = pb^{-1}$. Using the AG^{**} dentity and idempotent reassociation gives

$$ad^{-1} = a(pb^{-1}) = p(ab^{-1}) = (pa)b^{-1} = ab^{-1}.$$

By (iii) and the left-invertive law,

$$a = (ad^{-1})a = (aa)d^{-1}, \qquad (aa)a^{-1} = (a^{-1}a)a = pa = a.$$

Since $aa, d^{-1}, a^{-1} \in G_p$, injectivity of the left translation by aa gives $d^{-1} = a^{-1}$, hence $d = a$. Therefore $a = pb = (aa^{-1})b$, which is (iv). Finally $(iv) \Rightarrow (i)$ follows directly from the definition of the natural order. □

For a congruence ρ on S, put

$$\ker\rho = \bigcup_{e\in E(S)} e\rho, \qquad \operatorname{tr}\rho = \rho|_{E(S)}.$$

The first is the *kernel* and the second is the *trace*. Dudek and Gigoń proved that congruences are determined by these data: for a congruence ρ,

$$(a,b) \in \rho \iff (aa^{-1}, bb^{-1}) \in \operatorname{tr}\rho \quad \text{and} \quad ab^{-1} \in \ker\rho, \tag{1}$$

and consequently a congruence is uniquely determined by its kernel and trace [5, Theorem 4.2]. They also characterized abstract kernel–trace pairs that arise from congruences [5, Theorem 5.2]. These bilateral results are used below only after the one-sided theory has been identified with ordinary congruence theory.

3. Right congruences collapse to congruences

Theorem 3.1 (collapse theorem)**.** *Every right congruence on a completely inverse AG^{**}-groupoid is a congruence.*

Proof. Let $a\,\rho\,b$ and $c \in S$. Right compatibility gives

$$ac\,\rho\,bc.$$

Multiplying once more on the right by cc^{-1} gives

$$(ac)(cc^{-1})\,\rho\,(bc)(cc^{-1}).$$

By the left-invertive law and the inverse identity,

$$(ac)(cc^{-1}) = ((cc^{-1})c)a = ca, \qquad (bc)(cc^{-1}) = ((cc^{-1})c)b = cb.$$

Hence $ca \,\rho\, cb$. Thus ρ is left compatible as well as right compatible, and therefore is a congruence. □

Corollary 3.2. *If $RC(S)$ is the set of right congruences and $C(S)$ the congruence lattice, then*

$$RC(S) = C(S)$$

as ordered sets, and hence as complete lattices.

Corollary 3.3. *Every right congruence ρ on S is uniquely determined by $(\ker \rho, \operatorname{tr} \rho)$ and satisfies the criterion* (1).

Proof. Apply Theorem 3.1 and the bilateral kernel–trace theorem [5, Theorem 4.2]. □

Remark 3.4. Theorem 3.1 is the conceptual point that was missing from the original dissertation organization. Once it is stated first, right congruences no longer appear to form a larger class requiring an independent kernel–trace theory.

4. Pseudo-right and right congruence pairs

We now reconstruct the one-sided pair formalism from [1], using the corrected formulation established in the mathematical audit of Chapter 6.

Definition 4.1. Let K be a full subgroupoid of S and let τ be a congruence on $E(S)$. Here *full* means that $E(S) \subseteq K$. We call (K, τ) a *pseudo-right congruence pair* if

$$b \in K, \quad a \geqslant b, \quad aa^{-1} \,\tau\, bb^{-1} \implies a \in K. \tag{2}$$

It is a *right congruence pair* if, in addition, for every $a \in K$ there exists $b \in S$ such that

$$a \geqslant b, \quad aa^{-1} \,\tau\, bb^{-1}, \quad b^{-1} \in K. \tag{3}$$

For a pseudo-right congruence pair define $\rho_{(K,\tau)}$ by

$$\begin{aligned} a \,\rho_{(K,\tau)}\, b \iff & ab^{-1},\ a^{-1}b,\ ba^{-1},\ b^{-1}a \in K, \\ & (aa^{-1})(b^{-1}b) \,\tau\, aa^{-1} \,\tau\, bb^{-1}. \end{aligned} \tag{4}$$

Lemma 4.2 (collection in K)**.** *If $a\,\rho_{(K,\tau)}\,b$ and $b \in K$, then $a \in K$.*

Proof. From $a\,\rho_{(K,\tau)}\,b$ we have $ab^{-1} \in K$ and

$$(aa^{-1})(bb^{-1})\,\tau\,aa^{-1}\,\tau\,bb^{-1}.$$

Since $b \in K$ and K is a subgroupoid,

$$y = (ab^{-1})b \in K.$$

By the left-invertive law,

$$y = (bb^{-1})a \leqslant a.$$

Also

$$y^{-1} = ((ab^{-1})b)^{-1} = (a^{-1}b)b^{-1}.$$

Using the medial law,

$$\begin{aligned} yy^{-1} &= ((ab^{-1})b)((a^{-1}b)b^{-1}) = ((ab^{-1})(a^{-1}b))(bb^{-1}) \\ &= ((aa^{-1})(b^{-1}b))(bb^{-1}) = (aa^{-1})(bb^{-1}), \end{aligned}$$

where the last equality is computed in the semilattice $E(S)$. Thus

$$aa^{-1}\,\tau\,yy^{-1}.$$

Now $a \geqslant y$, $y \in K$, and (2) gives $a \in K$. □

Theorem 4.3 (pair-generated right congruence)**.** *Let (K, τ) be a pseudo-right congruence pair. Then $\rho_{(K,\tau)}$ is a right congruence. Its trace is τ, and its kernel is*

$$H = \left\{a \in K :\ \exists b \in S \text{ with } a \geqslant b,\ b^{-1} \in K,\ aa^{-1}\,\tau\,bb^{-1}\right\}. \tag{5}$$

Moreover, if $K_1 \subseteq K_2$ and $\tau_1 \subseteq \tau_2$, then

$$\rho_{(K_1,\tau_1)} \subseteq \rho_{(K_2,\tau_2)}.$$

Proof. Reflexivity follows from fullness, since $aa^{-1} = a^{-1}a \in E(S) \subseteq K$. Symmetry is built into the four membership conditions in (4) and into the symmetry of τ.

For right compatibility, assume $a\,\rho_{(K,\tau)}\,b$ and let $c \in S$. The product-inverse and medial laws give

$$\begin{aligned} (ac)(bc)^{-1} &= (ab^{-1})(cc^{-1}) \in K, \\ (ac)^{-1}(bc) &= (a^{-1}b)(c^{-1}c) \in K, \\ (bc)(ac)^{-1} &= (ba^{-1})(cc^{-1}) \in K, \\ (bc)^{-1}(ac) &= (b^{-1}a)(c^{-1}c) \in K. \end{aligned}$$

Here $cc^{-1} = c^{-1}c \in E(S) \subseteq K$. Put $e_a = aa^{-1}$, $e_b = bb^{-1}$ and $e_c = cc^{-1}$. From

$$e_a e_b \,\tau\, e_a \,\tau\, e_b$$

and compatibility of τ on the semilattice $E(S)$, multiplication by e_c yields

$$e_a e_b e_c \,\tau\, e_a e_c \,\tau\, e_b e_c.$$

Since $(ac)(ac)^{-1} = e_a e_c$ and $(bc)(bc)^{-1} = e_b e_c$, condition (4) holds for ac and bc. Hence $ac \,\rho_{(K,\tau)}\, bc$.

For transitivity, suppose

$$a \,\rho_{(K,\tau)}\, b, \qquad b \,\rho_{(K,\tau)}\, c.$$

Right compatibility applied to the first relation with c^{-1} gives

$$ac^{-1} \,\rho_{(K,\tau)}\, bc^{-1}.$$

Because $bc^{-1} \in K$, Lemma 4.2 yields $ac^{-1} \in K$. Similarly, multiplying the second relation by a^{-1} and using $ba^{-1} \in K$ gives $ca^{-1} \in K$. Since K is full,

$$(ca^{-1})(aa^{-1}) = a^{-1}c \in K, \qquad (ac^{-1})(cc^{-1}) = c^{-1}a \in K.$$

Thus all four membership conditions for $a \,\rho_{(K,\tau)}\, c$ hold. For the idempotent part, from $e_a \,\tau\, e_b \,\tau\, e_c$ and $e_b e_c \,\tau\, e_b$ we obtain

$$e_a e_c \,\tau\, e_b e_c \,\tau\, e_b \,\tau\, e_a,$$

so the remaining condition in (4) also holds. Thus $\rho_{(K,\tau)}$ is an equivalence relation and is right compatible.

If $e, f \in E(S)$, fullness supplies all membership conditions and

$$e \,\rho_{(K,\tau)}\, f \iff e \,\tau\, f,$$

so $\operatorname{tr} \rho_{(K,\tau)} = \tau$.

For the kernel, let first $a \in \ker \rho_{(K,\tau)}$. Then $a \,\rho_{(K,\tau)}\, e$ for some $e \in E(S)$. Lemma 4.2 gives $a \in K$. Put $b = ea$. Then $b \leqslant a$, and

$$b^{-1} = (ea)^{-1} = ea^{-1} \in K.$$

Moreover $bb^{-1} = e(aa^{-1})$, and the idempotent condition in $a \,\rho_{(K,\tau)}\, e$ yields $aa^{-1} \,\tau\, bb^{-1}$. Hence $a \in H$.

Conversely, let $a \in H$ and choose b as in (5). Put $f = bb^{-1} \in E(S)$. Since $b \leqslant a$, Lemma 2.1 gives $b = fa$. Hence

$$fa = b \in K, \qquad fa^{-1} = b^{-1} \in K.$$

Also $a, f \in K$ implies $af \in K$, and

$$(fa^{-1})(aa^{-1}) = a^{-1}f \in K.$$

Thus the four membership conditions for $a\,\rho_{(K,\tau)}\,f$ hold. From $aa^{-1}\,\tau\,f$ and compatibility of τ we obtain

$$(aa^{-1})f\,\tau\,aa^{-1}\,\tau\,f.$$

Therefore $a\,\rho_{(K,\tau)}\,f$, so $a \in \ker\rho_{(K,\tau)}$. This proves $\ker\rho_{(K,\tau)} = H$.

The monotonicity assertion follows directly from the defining conditions in (4). □

Theorem 4.4 (exact pair representation)**.** *If (K,τ) is a right congruence pair, then $\rho_{(K,\tau)}$ is a right congruence with*

$$\ker\rho_{(K,\tau)} = K, \qquad \operatorname{tr}\rho_{(K,\tau)} = \tau.$$

Proof. By Theorem 4.3, the kernel is the set H in (5), and $H \subseteq K$. If $a \in K$, condition (3) supplies $b \in S$ with $a \geqslant b$, $b^{-1} \in K$ and $aa^{-1}\,\tau\,bb^{-1}$. Hence $a \in H$, so $K \subseteq H$ and $H = K$. □

5. Every right congruence arises from a right pair

Lemma 5.1. *If ρ is a right congruence on S, then* $\ker\rho$ *is a full subgroupoid of S and* $\operatorname{tr}\rho$ *is a congruence on $E(S)$.*

Proof. By Theorem 3.1, ρ is a congruence. If $a\,\rho\,e$ and $b\,\rho\,f$ with $e, f \in E(S)$, then $ab\,\rho\,ef$, and $ef \in E(S)$. Hence $ab \in \ker\rho$. Also $E(S) \subseteq \ker\rho$. The trace is simply the restriction of a congruence to the subsemilattice $E(S)$. □

Theorem 5.2 (corrected converse theorem)**.** *Let ρ be a right congruence on S. Then*

$$(K,\tau) = (\ker\rho, \operatorname{tr}\rho)$$

is a right congruence pair and

$$\rho = \rho_{(K,\tau)}.$$

Proof. By Theorem 3.1, ρ is a congruence. Lemma 5.1 gives the structural requirements on K and τ.

To verify (2), let $b \in K$, $a \geqslant b$ and $aa^{-1}\,\tau\,bb^{-1}$. Then

$$aa^{-1}\,\rho\,bb^{-1}.$$

Right multiplication by a gives

$$(aa^{-1})a\,\rho\,(bb^{-1})a.$$

The left side is a, while $b \leqslant a$ and Lemma 2.1 give $(bb^{-1})a = b$. Thus $a\,\rho\,b$. Since $b \in K$, there is $e \in E(S)$ with $b\,\rho\,e$, so $a\,\rho\,e$ and $a \in K$.

For (3), let $a \in K$. By the standard kernel theory, the kernel of a congruence on a completely inverse AG**-groupoid is a full completely inverse AG**-subgroupoid, and hence is closed under taking inverses [5, Section 5]; see also [3]. Hence $a^{-1} \in K$. Taking $b = a$ gives

$$a \geqslant a, \qquad aa^{-1}\,\tau\,aa^{-1}, \qquad a^{-1} \in K.$$

Thus (K, τ) is a right congruence pair.

Theorem 4.4 shows that $\rho_{(K,\tau)}$ is a right congruence with kernel K and trace τ. By Theorem 3.1, both ρ and $\rho_{(K,\tau)}$ are ordinary congruences. The published kernel–trace uniqueness theorem [5, Theorem 4.2] therefore yields

$$\rho = \rho_{(K,\tau)}. \qquad \square$$

Remark 5.3 (removal of the old extra assumptions)**.** The original dissertation formulation of the converse theorem included the additional hypotheses

$$a = a^{-1}, \qquad (ae)a \in E(S).$$

They are not needed. Their role disappears once Theorem 3.1 is isolated and kernel–trace uniqueness is invoked at the end of the proof.

Corollary 5.4 (right-pair correspondence)**.** *The maps*

$$\rho \longmapsto (\ker\rho, \operatorname{tr}\rho), \qquad (K, \tau) \longmapsto \rho_{(K,\tau)}$$

are mutually inverse bijections between right congruences and right congruence pairs.

6. Comparison with standard congruence pairs

The collapse theorem makes it possible to compare the dissertation's one-sided pair formalism directly with the bilateral congruence-pair theory published later by Dudek and Gigoń [5]. Let $RP(S)$ denote the set of right congruence pairs of Definition 4.1, and let $CP(S)$ denote the standard congruence pairs in the sense of [5, Section 5].

Theorem 6.1 (equivalence of pair formalisms)**.** *For a completely inverse AG^{**}-groupoid S,*

$$RP(S) = CP(S)$$

as sets of kernel–trace data. For each common pair (K, τ), the right-pair relation (4) *coincides with the standard congruence determined by (K, τ).*

Proof. Let $(K, \tau) \in RP(S)$. By Theorem 4.4, $\rho_{(K,\tau)}$ is a right congruence with kernel K and trace τ. Theorem 3.1 makes it a congruence. The converse part of the standard congruence-pair theorem [5, Theorem 5.2] therefore implies that (K, τ) is a standard congruence pair. Hence $RP(S) \subseteq CP(S)$.

Conversely, let $(K, \tau) \in CP(S)$. The standard theorem produces a congruence ρ with kernel K and trace τ. It is, in particular, a right congruence, so Theorem 5.2 implies that (K, τ) is a right congruence pair. Thus $CP(S) \subseteq RP(S)$.

Finally, both constructions yield congruences having the same kernel and trace; uniqueness [5, Theorem 4.2] makes the two relations equal. □

Corollary 6.2 (simplified membership criterion)**.** *If (K, τ) is a right congruence pair, then the four membership conditions in* (4) *are equivalent, at the level of the resulting relation, to the standard kernel–trace criterion*

$$a \,\rho_{(K,\tau)}\, b \quad \Longleftrightarrow \quad (aa^{-1}, bb^{-1}) \in \tau \text{ and } ab^{-1} \in K.$$

Proof. By Theorem 6.1, $\rho_{(K,\tau)}$ is the unique ordinary congruence with kernel K and trace τ. Apply (1). □

Corollary 6.3 (order and lattice consequence)**.** *The kernel–trace map is an order isomorphism from $RC(S) = C(S)$ onto the ordered set of right congruence pairs (equivalently, standard congruence pairs) with coordinatewise inclusion. In particular,*

$$\rho_1 \subseteq \rho_2 \quad \Longleftrightarrow \quad \ker \rho_1 \subseteq \ker \rho_2 \text{ and } \operatorname{tr} \rho_1 \subseteq \operatorname{tr} \rho_2.$$

Proof. Inclusion of congruences implies inclusion of their kernels and traces. Conversely, coordinatewise inclusion of right pairs implies inclusion of the corresponding relations by Theorem 4.3. The bijection in Corollary 5.4 therefore preserves and reflects order. □

7. Discussion and provenance of the result

The mathematical effect of Theorem 3.1 is stronger than a simplification of one proof. It shows that one-sided compatibility carries no additional congruence information in the completely inverse AG** setting. Consequently, the quotient theory, the kernel–trace description, and the congruence-pair description of a right congruence are exactly those of an ordinary congruence.

This observation also clarifies the historical relationship between the dissertation construction and the later published bilateral theory. The pseudo-right/right pair machinery remains a valid intrinsic construction, and Theorems 4.3–5.2 provide a corrected archival version of it. At the same time, Theorem 6.1 shows that, once the ambient structure is assumed completely inverse, the same kernel–trace data are governed by the standard congruence-pair theory. Later work has continued to use congruence-pair and structural methods in this setting and in closely related AG** classes [6, 7, 10]. Thus the one-sided formulas are best understood not as a competing congruence theory, but as an alternative presentation that collapses to the bilateral one.

The source of the right-pair construction is Chapter 6 of the author's 1997 doctoral dissertation [1]. That chapter was not subsequently published as a journal article. The present version changes the mathematical organization in three essential ways: the collapse theorem is placed first; the converse representation theorem is stated without the unnecessary additional assumptions recorded in Remark 5.3; and the resulting pair theory is explicitly compared with the later published kernel–trace theory [4, 5]. This separation is important for the novelty claim: the bilateral kernel–trace theorem itself is prior published work, whereas the contribution here is the identification and corrected reduction of the right-congruence framework.

Conclusion

For completely inverse AG**-groupoids, right congruences are exactly congruences. Therefore every right congruence is uniquely determined by its

kernel and trace, and the lattice of right congruences is the ordinary congruence lattice. The corrected pseudo-right/right congruence-pair construction yields precisely the same kernel–trace data as the standard bilateral congruence-pair theory. In particular, the original four-membership relation can ultimately be replaced by the simpler kernel–trace criterion once the equivalence of pair formalisms has been established.

These results provide a clean theoretical endpoint for the unpublished right-congruence material and a stable algebraic basis for subsequent quotient-based applications.

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