Quasigroups and Related Systems **33** (2025), 209 – 226
https://doi.org/10.56415/qrs.v33.16

Duality for bounded distributive hyperlattice negatively ordered commutative monoids

Imane Douadi, $\boxed{\textit{Ali Oumhani}}$ *and Abdelaziz Amroune*

Abstract. The aim of this paper is to prove the Priestley representation theorem for bounded distributive lattices in the framework of bounded distributive hyperlattice negatively ordered commutative monoids. In this way, some results of Priestley's work are extended. In particular, we show that the dual of the category of Priestley spaces negatively ordered commutative semigroups is equivalent to the category of bounded distributive hyperattice negatively ordered commutative monoids.

1. Introduction

The idea of hyperstructures was first introduced by the French mathematician Frédéric Marty, in 1934, with the publication of the paper "Sur une généralisation de la notion de groupe" at the 8^{th} Congress of Scandinavian Mathematicians [20]. In this paper, Marty introduces the notion of hypergroups (or multigroups) from the analysis of their properties. However, due to his early death, Marty published only two papers related to his concept of hypergroups.

An operation is a relation that manipulates elements of a set and returns a value that is in another set. A hyperoperation is a generalization of an operation when it returns a set of values instead of a single value. The class of structures consisting by a set and at least one hyperoperation is called an algebraic hyperstructure. Hyperalgebras are a kind of hyperstructures as well as hypergroups, hyperrings, hyperlattices and so on. The hyperstructures theory has been studied from many points of view and applied

2010 Mathematics Subject Classification: 06B05, 06B15.
Keywords: Semigroup, ordred semigroup, lattice, homomorphism of hyperlattice, Priestley space, homomorphism of Priestley space, Priestley duality.
We acknowledge with thanks the support of the "Direction Générale de la Recherche Scientifique et du Développement Technologique (DGRSDT)", Algeria.

to several areas of mathematics, computer science and logic. In the field of the logic, multialgebras have been used as semantics for logical systems. Recently, matrix semantics based on multialgebras have been considered by Arnon Avron and his collaborators under the name of non-deterministic matrices (or Nmatrices) and used them to characterize logics, in particular, some paraconsistent logics in the class of Logics of Formal Inconsistency – **LFIs** (see for instance [4]), for more detail see [15]. Some applications can also be found in e.g. [8, 9, 11].

It is now a well-established branch of algebraic theory and has been used in various fields by many authors see, for example, [1, 2, 18, 19, 25, 26, 27] and books e.g. [10, 30].

The development of new results and concepts in the theory of hyperalgebras requires a careful survey of what has already been done in the literature on the subject and this task is not very easy due to the different approaches proposed in the literature to treat the same concept. For example, the hyperlattice, that is a lattice with hyperoperations, has been introduced by M. Benado under the name of multistructure [5], by J. Morgado [21] as reticuloide, and most of the authors use the terms hyperlattice or multilattice, for additional specifics see also [15] .

Koguep et al. [18], KonstantinidouMittas [19], respectively, introduced the notion of hyperlattices and studied ideals and filters in these structures. Prime ideals and prime filters in hyperlattices have been examined and studied by R. Ameri et al. [2]. Rasouli and Davvaz defined and gave a fundamental relation on a hyperlattice and obtained a lattice from a hyperlattice. Furthermore, they defined a topology on the set of prime ideals of a distributive hyperlattice [26, 27].

In 1937, the representation theorems that appeared by M. Stone [29] were proved that every Boolean algebra is isomorphic to a set of $\{I_a : a \in A\}$ (where I_a denotes the set of prime ideals of A not containing a). Since then, the representation theorem for distributive lattices has known a vast development. Birkhoff in [6]; asserts that any finite distributive lattice L is isomorphic to the lattice of the ideals of the partial order of the join-irreducible elements of L.

H. Priestley improved a kind of duality for bounded distributive lattices in the papers [23, 24]. These representation theorems enable a deep and concrete comprehension of the lattices as well as their structures. Topological duality for Boolean algebras [29] and distributive lattices [28] is a useful tools for studying relational semantics for propositional logic. Canonical extensions [12, 13, 14, 16, 17], provide a way of looking at these semantics algebraically. Our motivation finds its place in the following opinion:

"Stone's duality and its variants are central in making the link between syntactical and semantic approaches to logic, also in theoretical computer science, this link is central as the two sides correspond to specification languages and the space of computational states. This ability to translate faithfully between algebraic specification and spatial dynamics have often proven themselves to be a powerful theoretical tool as well as a handle for making practical problems decidable" see [14].

In this paper, we develop a representation theory for bounded distributive hyperlattice negatively ordered commutative monoids. In this way, we extend some results of [3, 23, 24], exactly, we give a representation theorem for bounded distributive hyperlattices negatively ordered commutative monoids.

This paper is organized as follows: In the next section, basic definitions and notions are presented. In the third section, we give and prove the main result. More precisely, we show that the dual of the category Priestley spaces negatively ordered commutative semigroups is equivalent to the *category of bounded distributive hyperattice negatively ordered commutative monoids*. Finally, we give some examples.

2. Basic concepts

In this section, we present several hyperlattice results that will help us build our study. Further information can be found in [1, 2, 3, 18, 19, 25, 26, 27].

Let A be a nonempty set. A *partially ordered semigroup* is a structure $(A, \cdot, \leqslant)$ satisfies the following condition: (i) $(A, \cdot)$ is a semigroup; (ii) $(A, \leqslant)$ is a partially ordered set; (iii) for all $a, b, x \in A$, $a \leqslant b$ implies $a \cdot x \leqslant b \cdot x$ and $x \cdot a \leqslant x \cdot b$. A partially ordered semigroup is said to be *positively* (*negatively*) *ordered* if $x \leqslant x \cdot y$ and $y \leqslant x \cdot y$ ($x \cdot y \leqslant x$ and $x \cdot y \leqslant y$) for all $x, y \in A$.

Let $(L, \leqslant)$ be a partially ordered set (poset). A subset E of L is said to be increasing (decreasing) if, whenever $x \in E$, $y \in L$ and $x \leqslant y$ ($y \leqslant x$), we have $y \in E$.

Let L be a nonempty set and $P^*(L)$ denotes the set of all nonempty subsets of L. Maps $h : L \times L \to P^*(L)$, are called *hyperoperations* [20].

Let L be a nonempty set, $\wedge$ be a binary operation and $\sqcup$ be a hyperoperation on L. L is called a *hyperlattice* if for all a,b,$c \in L$ the following conditions hold: (i) $a \in a \sqcup a$, and $a \wedge a = a$; (ii) $a \sqcup b = b \sqcup a$, and $a \wedge b = b \wedge a$; (iii) $a \in [a \wedge (a \sqcup b)] \cap [a \sqcup (a \wedge b)]$; (iv) $a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c$, and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$; (v) $a \in a \sqcup b \Rightarrow a \wedge b = b$.

A hyperlattice L with the property $a \wedge (b \sqcup c) \subseteq (a \wedge b) \sqcup (a \wedge c)$ is called *distributive*, where for all nonempty subsets A and B of L we define $A \wedge B = \{a \wedge b \mid a \in A, b \in B\}$ and $A \sqcup B = \cup\{a \sqcup b \mid a \in A, b \in B\}$.

The converse of condition (v) is true. Indeed using (iii), we obtain $a \in a \sqcup b$ by taking $b = a \wedge b$. Hence, we can define a partial order on L by: $a \leqslant b \Leftrightarrow b \in a \sqcup b \Leftrightarrow a \wedge b = a$.

A hyperlattice L is called *bounded* if there exist $0, 1 \in L$ such that for all $a \in L$, $0 \leqslant a \leqslant 1$. We say that 0 is the *least element* and 1 is the *greatest element*.

Consider a lattice $(L, \wedge, \vee)$. We define the *Nakano hyperoperation* $\sqcup$ on L by $x \sqcup y = \{z \in L \mid z \vee x = z \vee y = x \vee y\}$, for all x, $y \in L$. To the best of our knowledge, the $\sqcup$ hyperoperation was first introduced by Nakano in [22], which is an investigation of hyperrings. A non-empty subset I of a hyperlattice L is called a *hyperlattice ideal* if the following conditions hold: (i) if $x \in I$, $y \leqslant x$ and $y \in L$, then $y \in I$; (ii) if $x, y \in I$, then $x \sqcup y \subseteq I$. A non-empty subset F of a hyperlattice L is called a *hyperlattice filter* if the following conditions hold:(i) if $x \in F$, $x \leqslant y$ and $y \in L$, then $y \in F$;(ii) if $x, y \in F$, then $x \wedge y \in F$. A hyperlattice ideal or filter is called *proper* if it does not coincide with L.

2.1. A hyperlattice negatively ordered commutative monoids

In this subsection, we recall the notions of hyperlattice negatively ordered commutative monoid and we give some examples of this algebraic structure.

Definition 2.1. Let L be a non-empty set, $\wedge$, $(\cdot)$ are binary operations and $\sqcup$ is a hyperoperation on L. L is called *bounded hyperlattice ordered commutative monoid* if the following conditions hold:

(*i*) $(L, \wedge, \sqcup)$ is a hyperlattice;

(*ii*) $(L, \cdot, e)$ is a commutative monoid;

(*iii*) for all $a, b, c \in L : a \cdot (b \wedge c) \leqslant (a \cdot b) \wedge (a \cdot c)$ and $a \cdot (b \sqcup c) \subseteq (a \cdot b) \sqcup (a \cdot c)$.

The condition (*iii*) implies that for all $a, b, c \in L : a \leqslant b \Rightarrow a \cdot c \leqslant b \cdot c$.

A hyperlatttice ordered commutative monoid L with the property $a \wedge (b \sqcup c) \subseteq (a \wedge b) \sqcup (a \wedge c)$ is called *distributive*. Also, a hyperlattice ordered commutative monoid L is called bounded if there exist $0, 1 \in L$ such that for all $a \in L$, $0 \leqslant a \leqslant 1$. We say that 0 is the least element and 1 is the greatest element, and if $(L, \wedge, \sqcup, \cdot, 0, 1)$ is a bounded hyperlattice positively (negatively) ordered commutative monoid then $e = 0$ $(e = 1)$.

Example 2.2. Let $A = \{0, a, b, c, 1\}$ such that $0 < a < b < c < 1$ and

$\cdot$	0	a	b	c	1
0	0	a	b	c	1
a	a	b	c	1	0
b	b	c	1	0	a
c	c	1	0	a	b
1	1	0	a	b	c

$(A, \leqslant, 0, 1)$ is a bounded distributive lattice. Define on A the hyperoperation $\sqcup$ by $x \sqcup y = \{t \in A \mid t \geqslant x \vee y\}$. Then $(A, \wedge, \sqcup, \cdot, 0, 1)$ is a bounded distributive hyperlattice ordered commutative monoid.

Lemma 2.3. *Let $(L, \wedge, \vee, 0, 1)$ be a bounded distributive lattice. Then*

(*i*) $(L, \wedge, \sqcup, \cdot, 0, 1)$ *is a* bounded distributive hyperlattice positively ordered commutative monoid *where the opretions $\sqcup$ and $\cdot$ are given by* $a \sqcup b = \{x \in L \mid a \vee b = a \vee x = b \vee x\}$

$$a \cdot b = \begin{cases} a \vee b & \text{if } a = 0 \text{ or } b = 0; \\ 1 & \text{otherwise.} \end{cases}$$

for all a, $b \in L$.

(*ii*) $(L, \wedge, \sqcup, \cdot, 0, 1)$ *is a* bounded distributive hyperlattice negatively ordered commutative monoid *where the opretions $\sqcup$ and $\cdot$ are given by* $a \sqcup b = \{x \in L \mid a \vee b = a \vee x = b \vee x\}$,

$$a \cdot b = \begin{cases} a \wedge b & \text{if } a = 1 \text{ or } b = 1; \\ 0 & \text{otherwise.} \end{cases}$$

for all a, $b \in L$.

(*iii*) $(L, \wedge, \sqcup, \cdot, 0, 1)$ *is a* bounded distributive hyperlattice positively ordered commutative monoid *where the opretions $\sqcup$ and $\cdot$ are given by* $a \sqcup b = \{x \in L \mid x \geq a \vee b\}$,

$$a \cdot b = \begin{cases} a \vee b & \text{if } a = 0 \text{ or } b = 0; \\ 1 & \text{otherwise.} \end{cases}$$

for all a, $b \in L$.

Proof. The proof is straightforward. □

Lemma 2.4. *Assume that* $\mathbb{U}$ *be the boolean algebra* $\{0,1\}$. *Then* $(\mathbb{U}, \wedge, \sqcup, ., 0, 1)$ *is a bounded distributive hyperlattice negatively ordered commutative monoid, where*

$\wedge$	0	1
0	0	0
1	0	1

$\sqcup$	0	1
0	$\{0\}$	$\{1\}$
1	$\{1\}$	$\{0,1\}$

and

$\cdot$	0	1
0	0	0
1	0	1

2.2. Ideals and filters

In this subsection, we give the notions of ideals and filters in hyperlattice negatively ordered commutative monoid.

Definition 2.5. Let $(L, \wedge, \sqcup, \cdot)$ be a hyperlattice ordered commutative monoid. A non-empty subset I of L is called an *ideal* if it hold

(i) $x \in I$, $y \leqslant x \Rightarrow y \in I$;

(ii) x,$y \in I \Rightarrow x \sqcup y \subseteq I$;

(iii) $x \in I$ or $y \in I \Rightarrow x \cdot y \in I$;

for every $x, y \in L$.

A proper ideal I is called a *prime ideal* if (iv) $x \wedge y \in I \Rightarrow (x \in I$ or $y \in I)$ and (v) $x \cdot y \in I \ \Rightarrow (x \in I$ or $y \in I)$ for all $x, y \in L$.

Definition 2.6. Let $(L, \wedge, \sqcup, \cdot)$ be a hyperlattice ordered commutative monoid. A non-empty subset F of L is called a *filter* if it hold

(i) $x \in F$, $x \leqslant y \Rightarrow y \in F$;

(ii) $x, y \in F \Rightarrow x \wedge y \in F$;

(iii) $x, y \in F \Rightarrow x \cdot y \in F$;

for every $x, y \in L$.

A proper filter F is called a *prime filter* if (iv) $(x \sqcup y) \cap F \neq \emptyset \Rightarrow (x \in F$ or $y \in F)$; (v) $x \cdot y \in F \Rightarrow (x \in F$ or $y \in F)$.

Theorem 2.7. [2] *Let* $(L, \wedge, \sqcup)$ *be a hyperlattice. If* I *is a prime ideal of* L*, then* $L - I$ *is a prime filter of* L*. Similarly, if* F *is a prime filter of* L*, then* $L - F$ *is a prime ideal of* L*.*

The following theorem shows that the complement of a prime filter (ideal) is a prime ideal (filter).

Theorem 2.8. *Let $(L, \wedge, \sqcup, \cdot)$ be a hyperlattice negatively ordered commutative monoid. If I is a prime ideal of L, then $L - I$ is a prime filter of L. Similarly, if F is a prime filter of L, then $L - F$ is a prime ideal of L.*

Proof. First, we prove that $L - I$ is a prime filter of L. Let $x \in L - I$, $y \in L$ such that $x \leqslant y$, we show that $y \in L - I$. It is clear that $x \notin I$. Suppose $y \notin L - I$, so $y \in I$. We have $x \leqslant y$ and I is an ideal, therefore $x \in I$ that is a contradiction. So $y \in L - I$. Assume $x, y \in L - I$. We show that $x \wedge y \in L - I$. It is clear that $x, y \notin I$. If $x \wedge y \in I$, then $x \in I$ or $y \in I$ because I is a prime ideal, which is a contradiction, so $x \wedge y \in L - I$. Let $x, y \in L - I$. We show that $x \cdot y \in L - I$. Since L is negatively ordered it is clear that $x, y \notin I$. If $x \cdot y \in I$, then $x \in I$ or $y \in I$ because I is a prime ideal, which is a contradiction, so $x \cdot y \in L - I$. Hence, $L - I$ is a filter. It is enough to show that $L - I$ is a prime filter. Suppose $x, y \in L$ and $(x \sqcup y) \cap (L - I) \neq \emptyset$. So there exists $z \in L$ such that $z \in x \sqcup y$ and $z \in L - I$. If $x, y \notin L - I$, then $x, y \in I$, therefore $x \sqcup y \subseteq I$ because I is an ideal. Hence $(x \sqcup y) \cap (L - I) = \emptyset$, which is a contradiction. So $x \in L - I$ or $y \in L - I$. Let $x, y \in L$, such that $x \cdot y \in L - I$. If $x \notin L - I$ and $y \notin L - I$, then $x \in I$ and $y \in I$. Since I is an ideal, $x \cdot y \in I$. So $x \cdot y \notin L - I$, which is a contradiction. So $x \in L - I$ or $y \in L - I$. Thus $L - I$ is a prime filter. Similarly, we prove that if F is a prime filter if and only if $L - F$ is a prime ideal of L. □

Proposition 2.9. [3] *Let S be a non-empty subset of a hyperlattice ordered commutative monoid $(L, \wedge, \sqcup, \cdot)$, then the smallest hyperlattice filter containing S has the form*

$$\langle S \rangle = \{x \in L \mid a_1 \wedge ... \wedge a_n \leqslant x, \textit{ for some } a_1, .., a_n \in S\}.$$

If $S = \{a\}$, we write $\langle S \rangle = \uparrow a = \{x \in L \mid a \leqslant x\}$.

Proposition 2.10. *If δ is a hyperlattice filter of a distributive hyperlattice negatively ordered commutative monoid $(L, \wedge, \sqcup, \cdot)$, then the smallest hyperlattice ordered commutative monoid filter containing δ has the form*

$$[\delta) = \{x \in L \mid a_1 \cdot ... \cdot a_n \leqslant x, \textit{ for some } a_1, .., a_n \in \delta\}.$$

Proof. First, we prove that $[\delta)$ is non-empty. Let $a \in \delta$, since $a \leqslant a$, then $a \in [\delta)$, hence $[\delta) \neq \emptyset$. To prove that $[\delta)$ is a filter, let $x \in [\delta)$ and $y \in L$ such that $x \leqslant y$, then $a_1 \cdot ... \cdot a_n \leqslant x \leqslant y$, so $y \in [\delta)$. On the other hand, for every $x, y \in [\delta)$, there exist $a_1, a_2, ..., a_n, b_1, b_2, ..., b_m$ such that $a_1 \cdot a_2 \cdot ... \cdot a_n \leqslant x$

and $b_1 \cdot b_2 \cdot ... \cdot b_m \leqslant y$. Then $(a_1 \cdot ... \cdot a_n) \wedge (b_1 \cdot b_2 \cdot ... \cdot b_m) \leqslant x \wedge y$. Since L is negatively ordered so, $x \wedge y \in [\delta)$. Now let $x, y \in [\delta)$, then there exist $a_1, a_2, ..., a_n, b_1, b_2, ..., b_m$ such that $a_1 \cdot a_2 \cdot ... \cdot a_n \leqslant x$ and $b_1 \cdot b_2 \cdot ... \cdot b_m \leqslant y$, so $(a_1 \cdot a_2 \cdot ... \cdot a_n) \cdot y \leqslant x \cdot y$ and $(a_1 \cdot a_2 \cdot ... \cdot a_n) \cdot (b_1 \cdot b_2 \cdot ... \cdot b_m) \leqslant (a_1 \cdot a_2 \cdot ... \cdot a_n) \cdot y$. Then, $(a_1 \cdot a_2 \cdot ... \cdot a_n) \cdot (b_1 \cdot b_2 \cdot ... \cdot b_m) \leqslant x \cdot y$. Therefore, $x \cdot y \in [\delta)$.

Next, let $a \in \delta$, since $a \leqslant a$, we have $a \in [\delta)$. Then $\delta \subseteq [\delta)$.

Finally, suppose that F is a filter with $\delta \subseteq F$. Then for any $x \in [\delta)$, then there exist $a_1, a_2, .., a_n \in \delta$ such that $a_1 \cdot ... \cdot a_n \leqslant x$, then $x \in F$. Therefore $[\delta) \subseteq F$. $\square$

Theorem 2.11. *Let $(L, \wedge, \sqcup, \cdot)$ be a distributive hyperlattice negatively ordered commutative monoid, F be a filter and I be an ideal of L. If $F \cap I = \emptyset$, then there is a prime filter P such that $F \subseteq P$ and $P \cap I = \emptyset$.*

Proof. For the condition $(a \sqcup b) \cap P \neq \emptyset$, then $a \in P$ or $b \in P$ see [3].

Suppose that $a \cdot b \in P$, since L be a distributive hyperlattice negatively ordered commutative monoid then, $a \in P$ and $b \in P$. $\square$

Corollary 2.12. *Let $(L, \wedge, \sqcup, \cdot)$ be a distributive hyperlattice negatively ordered commutative monoid. If I is an ideal and $a \in L - I$, then there exists a prime filter P such that $a \in P$ and $P \cap I = \emptyset$.*

Proof. Let I be an ideal and take $F = \{x \in L - I \mid a \cdot ... \cdot a \leqslant x\}$, it follows $F \cap I = \emptyset$. It is easy to show that F is a filter. By Theorem 2.11, there is a prime filter P such that $F \subseteq P$ and $P \cap I = \emptyset$. $\square$

2.3. Hyperlattice ordered monoid homomorphisms

In this subsection, we introduce the notion of hyperlattice ordered commutative monoid homomorphism, and we give some properties of them that we needed in the sequel.

Definition 2.13. Let $(L, \wedge, \sqcup, \cdot)$ and $(L', \curlywedge, \Cup, \odot)$ be two hyperlattice ordered commutative monoids and $f : L \to L'$ be a mapping. f is said to be a *hyperlattice ordered commutative monoid homomorphism* if $f(x \wedge y) = f(x) \curlywedge f(y)$, $f(x \sqcup y) \subseteq f(x) \Cup f(y)$ and $f(x \cdot y) \leqslant f(x) \odot f(y)$, for all $x, y \in L$. If f is a bijection, then f is said to be a *hyperlattice ordered commutative monoid isomorphism*.

Proposition 2.14. *Let $(L, \wedge, \sqcup, \odot)$ be the hyperlattice ordered commutative monoid, $(\{0,1\}, \wedge, \sqcup, \cdot, 0, 1)$ be the bounded hyperlattice negatively ordered commutative monoid in Lemma 2.4 and F be a subset of L. If F is a prime filter, then there is a surjective hyperlattices negatively ordered commutative monoid homomorphism $f : L \to \{0,1\}$, such that $F = f^{-1}(\{1\})$.*

Proof. Set $f(X) = 1$ if $X \subseteq F$, and $f(X) = 0$ otherwise.

For the conditions $f(x \wedge y) = f(x) \wedge f(y)$ and $f(x \sqcup y) \subseteq f(x) \sqcup f(y)$ see [3].

For the third homomorphism axiom, we have $f(x) \cdot f(y) = 1 \Leftrightarrow (f(x) = 1$ and $f(y) = 1) \Leftrightarrow (x \in F$ and $y \in F) \Leftrightarrow x \odot y \in F \Leftrightarrow f(x \odot y) = 1$. Hence, $f(x) \cdot f(y) = f(x \odot y)$. □

Corollary 2.15. *Let L be a distributive hyperlattice negatively ordered commutative monoid. If $a, b \in L$ are such that $a \nleq b$ there is a prime filter F such that $a \notin F$ and $b \in F$.*

Proof. Taking $\downarrow b$ in Corollary 2.12, which is an ideal of a distributive hyperlattice negatively ordered commutative monoid and $a \notin \downarrow b$. □

3. Priestley-style topological representation theorem

An ordered topological space is a triple $(X, \tau, \leqslant)$ such that (X, τ) is a topological space and $(X, \leqslant)$ is a poset. A clopen set in a topological space is a set which is both open and closed. The ordered topological space is said to be totally disconnected if for every $x, y \in X$ such that $x \nleqslant y$ there exists an increasing τ-clopen U and a decreasing τ-clopen V such that $U \cap V = \phi$, with $x \in U$ and $y \in V$. A Priestley space is a compact totally disconnected ordered topological space.

Definition 3.1. [3] If $(A, \wedge, \sqcup, 0, 1)$ is a bounded distributive hyperlattice, then its dual space is defined to be $T(A) = (X, \tau, \leqslant)$, where X is the set of all homomorphisms from A onto $(\{0,1\}, \wedge, \sqcup, 0, 1)$ preserving 0 and 1 and $\leqslant$ is the partial order defined by $f \leqslant g$ in X if and only if $f(a) \leqslant g(a)$ for all $a \in A$, τ the product topology induced from that of $\{0.1\}^A$.

Proposition 3.2. [3] *If $(A, \wedge, \sqcup, 0, 1)$ is a bounded distributive hyperlattice, then its dual space $T(A) = (X, \tau, \leqslant)$ is a Priestley space.*

Inspired by the notion of Priestley spaces (see, [23, 24]), in the following we give some definitions and properties of *Priestley spaces negatively ordered commutative semigroups.*

Definition 3.3. *A* Priestley space negatively ordered commutative semigroups *is a quadruple* $(X, \tau, \leqslant, \star)$ *such that* $(X, \tau, \leqslant)$ *is a Priestley space and* $(X, \leqslant, \star)$ *a negatively ordered commutative semigroup.*

Definition 3.4. If $(A, \wedge, \sqcup, \cdot, 0, 1)$ is a *bounded distributive hyperlattice negatively ordered commutative monoid*, then its dual space is defined to be $T(A) = (X, \tau, \leqslant, \star)$, where X is the set of all homomorphisms from A onto $(\{0,1\}, \wedge, \sqcup, ., 0, 1)$ preserving 0 and 1 and $\leqslant$ is the partial order defined by $f \leqslant g$ in X if and only if $f(a) \leqslant g(a)$ for all $a \in A$, τ the product topology induced from that of $\{0,1\}^A$ and $\star$ is the semigroup operation defined in X by

$$(f \star g)(a) = \begin{cases} f(a) \wedge g(a) & \text{if } f(a) \vee g(a) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

for all $a \in A$.

Proposition 3.5. *If* $(E, \wedge, \sqcup, \cdot, 0, 1)$ *is a bounded distributive hyperlattice negatively ordered commutative monoid, then* $T(E) = (X, \tau, \leqslant, \star)$ *is a Priestley space negatively ordered commutative semigroup.*

Proof. Suppose that $T(E) = (X, \tau, \leqslant, \star)$ is the dual space of $(E, \wedge, \sqcup, \cdot, 0, 1)$. We know that $T(E) = (X, \tau, \leqslant)$ is Priestey space see [3], it remains to show that the binary operation $\star$ on X makes $(X, \leqslant, \star)$ a negatively ordered commutative semigroup. Indeed, by a routine calculation we can show that $f \star g$ is an homomorphisme. It is clearly that $f \star g = g \star f$ for all $f, g \in X$. It is easy to show that $(f \star g) \star h = f \star (g \star h)$. So $(X, \star)$ is a commutative semigroup. Clearly that $(X, \leqslant)$ is a partially ordered set, where $f \leqslant g \Leftrightarrow f(x) \leqslant g(x)$ for all $x \in E$. Now, let $f, g, h \in X$ with $f \leqslant g$, then $f(x) \leqslant g(x)$, so $f(x) \wedge h(x) \leqslant g(x) \wedge h(x)$ and $h(x) \wedge f(x) \leqslant h(x) \wedge g(x)$. Hence $f \leqslant g$ implies $f \star h \leqslant g \star h$ and $h \star f \leqslant h \star g$ and $(X, \star, \leqslant)$ is a partially ordered semigroup, it remains to show that $(X, \star, \leqslant)$ is negatively ordered i.e., $f \star g \leqslant f$ and $f \star g \leqslant g$, which is direct, indeed, $(f \star g)(x) = f(x) \wedge g(x) \leqslant f(x)$, hence $f \star g \leqslant f$ and $(f \star g)(x) = f(x) \wedge g(x) \leqslant g(x)$, hence $f \star g \leqslant g$. And $T(E) = (X, \tau, \leqslant, \star)$ is a Priestley space negatively ordered commutative semigroup. □

In the following, we give a definition of *Priestley space negatively ordered commutative semigroup homomorphism.*

Definition 3.6. Let $(X, \tau, \leqslant, \star)$ and $(X', \tau', \preceq, \circledast)$ be two a *Priestley spaces negatively ordered commutative semigroups.* Then $f : X \to X'$ is called

1. *increasing* if for all $x, y \in X$, $x \leqslant y \Rightarrow f(x) \preceq f(y)$.
2. a *Priestley space negatively ordered commutative semigroup homomorphism* if is increasing, continuous and for all $x, y \in X$, $f(x \star y) \leqslant f(x) \circledast f(y)$.

If f is a bijection, then f is said to be a *Priestley space negatively ordered commutative semigroup isomorphism.*

In what follows, we will give necessary conditions that the dual of a *Priestley space negatively ordered commutative semigroup* be a *bounded distributive hyperlattice negatively ordered commutative monoid.*

Lemma 3.7. *Let $\delta = (X, \tau, \leqslant, \star)$ be a Priestley space negatively ordered commutative semigroup, then there exist a binary hyperoperation $\sqcup$ and a binary operation $\odot$ such that $(L(\delta), \cap, \sqcup, \odot, \emptyset, X)$ is a bounded distributive hyperlattice negatively ordered commutative monoid, where*

$$L(\delta) = \{Y \subseteq X \mid Y \textit{ is increasing and } \tau\textit{-clopen}\}$$

and $\sqcup, \odot$ are defined by

$$A \sqcup B = \{X \in L(\delta) \mid A \cup B = A \cup X = B \cup X\},$$
$$A \odot B = \begin{cases} A \cap B \textit{ if } \; A = X \textit{or } B = X; \\ \emptyset \textit{ otherwise.} \end{cases}$$

for all $A, B \in L(\delta)$.

Proof. Since $(L(\delta), \cap, \cup, \emptyset, X)$ is a bounded distributive lattice, then we have $(L(\delta), \cap, \sqcup, \emptyset, X)$ is a bounded distributive hyperlattice, and by Definition 2.1 $(L(\delta), \cap, \sqcup, \odot, \emptyset, X)$ is a bounded distributive hyperlattice negatively ordered commutative monoid. $\square$

In the following lemma, we show that the bidual of a bounded distributive hyperlattice negatively ordered commutative monoid is a bounded distributive hyperlattice negatively ordered commutative monoid.

Lemma 3.8. *Let $(E, \wedge, \sqcup, \cdot, 0, 1)$ be a bounded distributive hyperlattice negatively ordered commutative monoid. Then $(L(T(E)), \cap, \sqcup, \odot, \phi, X)$ a bounded distributive hyperlattice negatively ordered commutative monoid.*

Proof. We have $L(T(E)) = \{Y \subseteq X \mid Y \text{ is increasing and } \tau\text{-clopen}\}$. $\sqcup, \odot$ are defined by $A \sqcup B = \{C \in L(T(E)) \mid A \cup B = A \cup C = B \cup C\}$,

$$A \odot B = \begin{cases} A \cap B \text{ if } A = X \text{ or } B = X; \\ \phi \text{ otherwise.} \end{cases}$$

for all $A, B \in L(T(E))$. Hence, by Definition 2.1 $(L(T(E)), \cap, \sqcup, \odot, \phi, X)$ a bounded distributive hyperlattice negatively ordered commutative monoid. $\square$

Lemma 3.9. *Let* $(E, \wedge, \sqcup, \cdot, 0, 1)$ *be a bounded distributive hyperlattice negatively ordered commutative monoid, then the map* $\varphi_E : E \longrightarrow L(T(E))$ *defined by* $\varphi_E(a) = \{f \in X \mid f(a) = 1\}$ *is a hyperlattice negatively ordered commutative monoid isomorphism.*

Proof. Since E is a hyperlattice then φ_E is an isomorphism of hyperlattices see [3]. $L(T(E))$ is hyperlattice negatively ordered commutative monoid by Lemma 3.8. For the condition $\varphi_E(a \cdot b) \subseteq \varphi_E(a) \odot \varphi_E(b)$, for all a, $b \in E$ and $f \in X$; $f \in \varphi_E(a \cdot b) \Rightarrow f(a \cdot b) = 1 \Rightarrow f(a)f(b) = 1 \Rightarrow (f(a) = 1$ and $f(b) = 1) \Rightarrow f \in \varphi_E(a) \cap \varphi_E(b) \Rightarrow f \in \varphi_E(a) \odot \varphi_E(b)$. So $\varphi_E(a \cdot b) \subseteq \varphi_E(a) \odot \varphi_E(b)$ Therefore, φ_E is a isomorphism of E onto $L(T(E))$. $\square$

Lemma 3.10. *Let* $f : E_1 \to E_2$ *be a hyperlattice negatively ordered commutative monoid homomorphism, then the map* $T(f)(g) : g \mapsto g \circ f$ *of* $T(E_2)$ *onto* $T(E_1)$ *is a homomorphism of a Priestley space negatively ordered commutative semigroup.*

Proof. Firstly, $T(f)$ is increasing and continuous see [3, 23, 24]. Secondly, for all $g_1, g_2 \in T(E_1)$,

(1) If $g_1 \circ f(a = 1$, then $(T(f)(g_1) \cdot T(f)(g_2))(a) = g_2 \circ f(a)$, since $g_1 * g_2 \leqslant g_2$, we have $T(f)(g_1) \cdot T(f)(g_2) \geq ((g_1 * g_2) \circ f)(a) = T(f)(g_1 * g_2)(a)$.

(2) The case $g_2 \circ f(a) = 1$, similarly to case (1).

(3) If $g_1 \circ f(a) = 0$ and $g_2 \circ f(a) = 0$, then $T(f)(g_1) \cdot T(f)(g_2) = T(f)(g_1 * g_2)(a) = 0$.

Hence $T(f)$ is a Priestley space negatively ordered commutative semigroup homomorphism. $\square$

Lemma 3.11. *Let* $(P_1, \tau, \leqslant, \star)$, $(P_2, \tau', \leqslant, \circledast)$ *be a Priestley space negatively ordered commutative semigroups and let* $h : P_1 \to P_2$ *be a Priestley space negatively ordered commutative semigroup homomorphism. Then the map* $L(h) : y \mapsto h^{-1}(y)$ *is a homomorphism of* $L(P_2)$ *onto* $L(P_1)$.

Proof. Setting $L(P_1) = \{Z \subseteq X \mid Z$ is increasing and τ-clopen$\}$ and $L(P_2) = \{Z \subseteq Y \mid Z$ is increasing and τ-clopen$\}$. For all $y \in L(P_2)$ we have $L(h)(y) \in L(P_1)$. For all $y, z \in L(P_2)$ since h^{-1} commutes with set-theoretical operations we have, $L(h)(y \cap z) = L(h)(y) \cap L(h)(z)$ and $L(h)(y \sqcup z) \subseteq L(h)(y) \sqcup L(h)(z)$ (see [3]).

For the third homomorphism axiom, we have three cases:

(1) if $y = Y$, then $L(h)(y \circledast z) = L(h)(z) = X \star L(h)(z) = L(h)(y) \star L(h)(z)$.

(2) if $z = Y$, similarly to case (1).

(3) if $y \neq Y$ and $z \neq Y$, then $L(h)(y \circledast z) = \phi \subseteq L(h)(y) \star L(h)(z)$.

Hence, $L(h)$ is hyperlattice ordered commutative monoid homomorphism. □

Theorem 3.12. *Let $f : E_1 \to E_2$ be a hyperlattice negatively ordered commutative monoid homomorphism. Then $L(T(f)) \circ \varphi_{E_1} = \varphi_{E_2} \circ f$.*

Proof. The proof is similar to [7, Lemma 3.11]. □

Lemma 3.13. *Let $P = (X, \tau, \leqslant, \star)$ be a Priestley space negatively ordered commutative semigroup. Then the map $\pi_P : P \to T(L(P))$ defined by*

$$\pi_P(x)(Y) = \begin{cases} 1 \ \textit{if } x \in Y, \\ 0 \ \textit{if } x \notin Y. \end{cases}$$

for all $Y \in L(P)$, is a Priestley space negatively ordered commutative semigroup isomorphism.

Proof. Since $P = (X, \tau, \leqslant, \star)$ be a Priestley space negatively ordered commutative semigroup, and by Lemma 3.7 $L(P)$ is a bounded distributive hyperlattice negatively ordered commutative monoid. By Lemma 3.8 $T(L(P))$ is a Priestley space negatively ordered commutative semigroup. π_P is increasing, continuous and bijective see [3, 23, 24]. Let $x, y \in X$, if $\pi_P(x \star y)(Y) = 0$, then $\pi_P(x \star y)(Y) \leqslant \pi_P(x)(Y) \cdot \pi_P(y)(Y)$.

If $\pi_P(x \star y)(Y) = 1$, hence $x \star y \in Y$, since $P = (X, \tau, \leqslant, \star)$ a Priestley space negatively ordered commutative semigroup, and Y is increasing τ-clopen then $x \in Y$ and $y \in Y$, it follows that $\pi_P(x)(Y) = 1$ and $\pi_P(y)(Y) = 1$, so $\pi_P(x)(Y) \cdot \pi_P(y)(Y) = 1$, therefore $\pi_P(x \star y)(Y) \leqslant \pi_P(x)(Y) \cdot \pi_P(y)(Y)$. Then, π_P is a Priestley space negatively ordered commutative semigroup isomorphism. □

Theorem 3.14. *Let $h : P_1 \to P_2$ be a Priestley space negatively ordered commutative semigroup isomorphism. Then $T(L(h)) \circ \pi_{P1} = \pi_{P2} \circ h$.*

Proof. Similarly to [7, Lemma 3.12]. □

In the following result, we give a Priestley duality for a *bounded distributive hyperlattices negatively ordered commutative monoids.*

Theorem 3.15. *The dual of the category of a Priestley spaces negatively ordered commutative semigroups is equivalent to the category of bounded distributive hyperlattices negatively ordered commutative monoids.*

Proof. By Lemma 3.9, Lemma 3.13, Theorem 3.12 and Theorem 3.14. □

4. Examples

The following example illustrates the fact that there is a isomorphism between a *bounded hyperlattice negatively commutative monoid* and their bidual.

Example 4.1. $(D(12), \wedge, \vee, 1, 12)$, where $D(12)$ is the set of positive divisors of 12, so $(D(12), \wedge, \vee, 1, 12)$ is a bounded lattice with $x \wedge y$ and $x \vee y$ as the greatest common divisor and the least common multiplier of x and y.

Define on $D(12)$ the hyperoperation $\sqcup$ and the binary operation $(\cdot)$ by: $x \sqcup y = \{z \in D(12) \mid z \vee x = z \vee y = x \vee y\}$;

$$x \cdot y = \begin{cases} x \wedge y & \text{if } x \vee y = 12, \\ 0 & \text{otherwise.} \end{cases}$$

for all $x, y \in D(12)$. Then by Lemma 2.3 (ii) $(D(12), \wedge, \sqcup, \cdot, 1, 12)$ is a bounded hyperlattice negatively commutative monoid. $T(D(12))$ is the set of homomorphisms from $D(12)$ onto $\{0, 1\}$, hence $T(D(12)) = \{f_1, f_2, f_3\}$ where f_1, f_2, f_3 are given by the table:

$D(12)$	1	2	3	4	6	12
f_1	0	0	0	1	0	1
f_2	0	1	0	1	1	1
f_3	0	0	1	0	1	1

Its bidual is $L(T(D(12))) = \{\emptyset, \{f_2\}, \{f_3\}, \{f_1, f_2\}, \{f_2, f_3\}, X\}$, where $X = \{f_1, f_2, f_3\}$. Then $(L(T(D(12))), \cap, \sqcup, \odot, \emptyset, X)$ is a bounded distributive hyperlattice ordered commutative monoid, where

$$x \sqcup y = \{C \in L(T(D(12))) \mid A \cup B = C \cup A = C \cup B\}$$

and

$$A \odot B = \begin{cases} A \cap B & \text{if } A = X \text{ or } B = X, \\ \phi & \text{otherwise.} \end{cases}$$

for all $A, B \in L(T(D(12)))$. Finally, $\varphi_{D(12)} : A \to L(T(D(12)))$ is given by the table:

$D(12)$	1	2	3	4	6	12
$\varphi_{D(12)}(a_i) \mid i = 1$ to 6	$\emptyset$	$\{f_2\}$	$\{f_3\}$	$\{f_1, f_2\}$	$\{f_2, f_3\}$	X

is a bounded distributive hyperlattice ordered commutative monoid isomorphism.

The following example shows that there is a isomorphism between a Priestley space negatively ordered commutative semigroup and its bidual.

Example 4.2. Let $(X, \tau, \leqslant, \star)$ be a Priestley space negatively ordered commutative semigroup, where $X = \{a, b, c, d\}$ and $a \leqslant b \leqslant c$, $a \leqslant b \leqslant d$ and

$\star$	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	b
d	a	b	b	d

and $L(X) = \{\phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}, X\}$, where the hyperoperation $\sqcup$ is given by:

$\sqcup$	$\emptyset$	$\{c\}$	$\{d\}$
$\emptyset$	$\emptyset$	$\{c\}$	$\{d\}$
$\{c\}$	$\{c\}$	$\{\emptyset, \{c\}\}$	$\{c, d\}$
$\{d\}$	$\{d\}$	$\{c, d\}$	$\{\emptyset, \{d\}\}$
$\{c, d\}$	$\{c, d\}$	$\{\{d\}, \{c, d\}\}$	$\{\{c\}, \{c, d\}\}$
$\{b, c, d\}$	$\{b, c, d\}$	$\{b, c, d\}$	$\{b, c, d\}$
X	X	X	X

$\sqcup$	$\{c, d\}$	$\{b, c, d\}$	X
$\emptyset$	$\{c, d\}$	$\{b, c, d\}$	X
$\{c\}$	$\{\{d\}, \{c, d\}\}$	$\{b, c, d\}$	X
$\{d\}$	$\{\{c\}, \{c, d\}\}$	$\{b, c, d\}$	X
$\{c, d\}$	$\{\emptyset, \{b, c\}\}$	$\{b, c, d\}$	X
$\{b, c, d\}$	$\{b, c, d\}$	$\{\phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}\}$	X
X	X	X	$L(X)$

and the binary operation $\odot$ is given by:

$\odot$	$\emptyset$	$\{c\}$	$\{d\}$	$\{c,d\}$	$\{b,c,d\}$	X
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{c\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{c\}$
$\{d\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{d\}$
$\{c,d\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{c,d\}$
$\{b,c,d\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{b,c,d\}$
X	$\emptyset$	$\{c\}$	$\{d\}$	$\{c,d\}$	$\{b,c,d\}$	X

and $(T(L(X)) = \{f_1, f_2, f_3, f_4\}$ where the homomorphisms f_1, f_2, f_3, f_4 are given by the table:

$L(X)$	$\emptyset$	$\{c\}$	$\{d\}$	$\{c,d\}$	$\{b,c,d\}$	X
f_1	0	0	0	0	0	1
f_2	0	0	0	0	1	1
f_3	0	0	1	1	1	1
f_4	0	1	0	1	1	1

The map $\pi_X : X \longrightarrow T(L(X)$ is defined by $\pi_X(a) = f_1$, $\pi_X(b) = f_2$, $\pi_X(c) = f_3$, $\pi_X(d) = f_4$ is a Priestley space negatively ordered commutative semigroup isomorphism.

5. Conclusion and future researcsh

The purpose of this paper is to establish a duality for bounded distributive hyperlattice negatively ordered commutative monoids. In this way, some results from [3, 23, 24] are extended. The main theorem shows that the dual of the category of a Priestley space negatively ordered commutative semigroups is equivalent to the category of bounded distributive hyperlattice negatively ordered commutative monoids.

For further investigation, we ask the following open question: It is possible to obtain such representation if we change the category of bounded distributive hyperattice negatively ordered commutative monoids by the category of bounded distributive hyperattice negatively ordered commutative hypermonoids.

References

[1] **R. Ameri, M. Amiri-Bideshki, S. Hoskova-Mayerova and A. B. Saeid**, *Distributive and Dual Distributive Elements in Hyperlattices*, An. Științ. Univ. "Ovidius" Constanța, Ser. Mat. **25** (2017), no. 3, 25 – 36.

[2] **R. Ameri, M. Amiri-Bideshki, A. B. Saeid and S. Hoskovamayerova**, *Prime filters of hyperlatices*, An. Univ. Ovidius Constanta Ser. Mat. **24** (2016), no. 2, 15 – 26.

[3] **A. Amroune and A. Oumhani**, *A representation theorem for bounded distributive hyperlattices*, Quasigroups Relat. Syst. , **26** (2018), 1 – 12.

[4] **A. Avron**, *Non-deterministic matrices and modular semantics of rules*, Logica Universalis: Towards a General Theory of Logic. Basel: Birkhäuser Basel, 2005, 149 – 167.

[5] **M. Benado**, *Asupra unei generalizări a noţiunii de structură*,Acad RP Roumania, Bul. Şti. Secţ. Şti. Mat. Fiz. **5** (1953), 41 – 48.

[6] **G. Birkhoff**, *Lattice theory*, Colloquium Publications, Third Edition, American Mathematical Society, RI, 1967.

[7] **V. Boicescu, A. Filipoiu, G. Georgescu and S. Rudeanu**, *Lukasiewicz-Moisil Algebras*, North Holland, Amsterdam 1991.

[8] **J. Chvalina, S. Hoskova-Mayerova and A. Deghan Nezhad**, *General actions of hypergroups and some applications*, An. Stiint. Univ. Ovidius Constanta Ser. Mat. **21** (2013), no. 1, 59 – 82.

[9] **J. Chvalina and S. Hoskova-Mayerova**, *On certain proximities and preorderings on the transposition hypergroups of linear first-order partial differential operators*, An. Stiint. Univ. Ovidius Constanta Ser. Mat. **22** (2014), no. 1, 85 – 103.

[10] **P. Corsini and V. Leoreanu**, *Applications of Hyperstructure Theory*, Kluwer Academic Publishers, Dordrecht, Hardbound 2003.

[11] **I. Cristea**, *Regularity of Intuitionistic Fuzzy Relations on Hypergroupoids*, An. Stiint. Univ. Ovidius Constanta Ser. Mat. **22** (2014), no. 1, 105 – 119.

[12] **M. Gehrke and J. Harding**, *Bounded lattice expansions*, J. Algebra. **238** (2001), no. 1, 345 – 371.

[13] **M. Gehrke and B. Jónsson**, *Bounded distributive lattices with operators*, Math. Japon. **40** (1994), no. 2, 207 – 215.

[14] **M. Gehrke**, *Stone duality, topological algebra, and recognition*, J. Pure Appl. Algebra. **220** (2016), no. 7, 2711 – 2747.

[15] **A. C. Golzio**, *A brief historical Survey on hyperstructures in Algebra and Logic*, S. Am. J. Log. **4** (2018), 91 – 119.

[16] **B. Jónsson**, *Algebras whose congruence lattices are distributive*, Math. Scand. **521** (1967), 110 – 121.

[17] **B. Jónsson and A. Tarski**, *Boolean Algebras with Operators*: part I, Am. J. Math. **73** (1951), no. 4, 891 – 939.

[18] **B.B.N. Koguep, C. Nkuimi and C. Lele**, *On fuzzy ideals of hyperlattice*, Int. J. Algebra. **2** (2008), no. 15, 739 – 750.

[19] **M. Konstantinidou and J. Mittas**, *An introduction to the theory of hyperlattices*, Math. Balcanica. **7** (1977), 187 – 193.

[20] **F. Marty**, *Sur une generalization de la notion de groups*, 8th Congress Math. Scandenaves, Stockholm. (1934), 45 – 49.

[21] **J. Morgado**, *Introdução à teoria dos reticulados*, Recife, Instituto de Física e Matemática, Universidade do Recife, 1962. (Introdução à teoria dos reticulados, 10).

[22] **T. Nakano**, *Rings and partly ordered systems*, Math. Z. **99** (1967), 355 – 376.

[23] **H.A. Priestley**, *Representation of distributive lattices by means of ordered Stone spaces*, Bull. Lond. Math. Soc. **2** (1970), 186 – 190.

[24] **H.A. Priestley**, *Ordered topological spaces and the presentation of distributive lattices*, Proc. Lond. Math. Soc. **24** (1972), no. 3, 507 – 530.

[25] **A. Rahnemai-Barghi**, *The prime ideal theorem for distributive hyperlattices*, Ital. J. Pure Appl. Math. **10** (2010), 75 – 78.

[26] **S. Rasouli and B. Davvaz**, *Lattice derived from hyperlattices*, Commun. Algebra. **38** (2010), 2720 – 2737.

[27] **S. Rasouli and B. Davvaz**, *Construction and spectral topology on hyperlattice*, Mediterr. J. Math. **7** (2010), 249 – 262.

[28] **M.H. Stone**, *The theory of representation for Boolean algebras*, Trans. Amer. Math. Soc. **40** (1936), no. 1, 37 – 111.

[29] **M.H. Stone**, *Topological representations of distributive lattices and Brouwerian logics*, Časopis Pěst. Math. **67** (1937), 1 – 25.

[30] **T. Vougiouklis**, *Hyperstructures and their Representations*, Hadronic Press, Inc. 1994.

Received June 14, 2024

Laboratory of pure and applied mathematics, University of M'sila
University Pole, Road Bordj Bou Arreridj, 28000 M'sila, Algeria

E-maila: imane.douadi@univ-msila.dz, abdelaziz.amroune@univ-msila.dz