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On fully dense acts

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Abstract. Over a monoid S fully dense S-acts as a generalization of monoids for which any right ideal is a generator are introduced and investigated. First some general properties on fully dense S-acts are mentioned and then monoids over which all free (projective) S-acts are fully dense are characterized. Moreover the relation between these kinds of S-acts and some other classes of S-acts such as prime and strongly prime S-acts is studied.

1. Introduction

In this paper, S is a monoid and an S-act A_S (or A) is a unitary right S-act. In [5] some classes of monoids in which every right ideal is a generator are studied and investigated. As a generalization of these monoids, in this paper we define the concept of fully dense S-acts. Let A be a right S-act and let B be a subact of A. From [5], B is said to be a *dense* subact of A if the trace of B in A is equal to A, i.e., $\mathrm{Tr}(B,A) := \bigcup_{\varphi \in \mathrm{Hom}(B,A)} \varphi(B) = A$ where Hom(B,A) denotes the set of homomorphisms from A to B. Also A is called *fully dense* if any subact of A is dense. In Section 2 of this paper, we study some properties of fully dense acts. Moreover in Section 3, as a generalization of strongly faithful S-acts, we introduce the notion of strongly prime S-acts and several equivalent conditions to being strongly prime are given. We study the interrelationship between (strongly) prime and fully dense acts. It is shown that over a commutative monoid S an S-act A is (strongly) prime if and only if A is contained in any fully invariant subact of $E(A)$, where $E(A)$ is the injective envelope of A.

Recall that an S-act A is called *injective* if for any S-act B, any subact C of B and any homomorphism $f : C \longrightarrow A$, there exists a homomorphism $\bar{f} : B \longrightarrow A$ such that $\bar{f}\mid_C = f$ (see [3]). Also the S-act A is called *quasi-injective* (*cyclic quasi-injective*) if it is injective relative to all inclusions

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from its subacts (cyclic subacts). For the sake of simplicity, we denote "cyclic quasi-injective", by "CQ-injective". Moreover an S-act A is called *projective* if for any S-acts D, C, every homomorphism $f : A \longrightarrow D$ can be lifted with respect to any epimorphism $h : C \longrightarrow D$, i.e., there exists a homomorphism $g : A \longrightarrow C$ such that $f = hg$.

An element $\theta \in A$ is called a zero element if $\theta s = \theta$ for every $s \in S$. Moreover the one element act is denoted by $\Theta = \{\theta\}$. Recall that an S-act is called *simple* if it contains no subacts other than itself. For an S-act A, by $E(A)$ and $Z(A)$, we mean the injective envelope and the zero elements of A respectively. For an S-act A, an equivalence relation ρ on A is called a *congruence* on A, if $a \ \rho \ b$ implies $as \ \rho \ bs$ for $a, b \in A$ and $s \in S$ ([3]). We denote the set of all congruences on A by $\mathrm{Con}(A)$. Also for an S-act A, by Δ_A and $\bigtriangledown_A$ we mean the congruences $\{(a, a) : a \in A\}$ and $A \times A$, respectivley. For a thorough account on the preliminaries, the reader is referred to [3].

2. Fully dense acts

In this section, we introduce the notions of fully dense acts and cyclically fully dense acts. We also bring out preliminary and basic properties of fully dense acts. From [3], recall that the trace of an S-act B in an S-act A is defined by $\mathrm{Tr}(B, A) := \bigcup_{\varphi \in \mathrm{Hom}(B,A)} \varphi(B)$. Also recall that an S-act G is a *generator* if S_S is a retract of G or equivalently, $\mathrm{Tr}(G, S) = S$ (see Theorem 2.3.16 of [3]).

Definition 2.1. Let S be a monoid and $B \leq A$ be S-acts. Then

(*i*) B is called a *dense* subact of A and is denoted by $B \subseteq_d A$, if $\mathrm{Tr}(B, A) = A$ (see [5]).

(*ii*) The S-act A is called (*cyclically*) *fully dense* if every (cyclic) subact of A is a dense subact. In particular if the right S-act S_S is a (cyclically) fully dense act, then we say that S is a (cyclically) *fully dense monoid.*

It is clear that for a non-trivial right ideal I of a monoid $S, I \subseteq_d S$ if and only if I is a generator. In [5], some classes of fully dense monoids are investigated and their structures have been determined. Trivially, if Θ is a subact of an S-act A, then A is a cyclically fully dense act if and only if $A =$

$Z(A)$ and so every (cyclically) fully dense monoid S which contains a left zero is trivial. The following proposition includes some general properties of fully dense acts.

Proposition 2.2. *The following hold for a monoid S.*

(i) *Let $\{A_i\}_{i\in I}$ be a family of S-acts. If $\coprod_{i\in I} A_i$ is a (cyclically) fully dense act, then each A_i is a (cyclically) fully dense act.*

(ii) *(cyclically) fully dense acts are preserved under products.*

(iii) *If $\{A_i\}_{i\in I}$ is a family of (cyclically) fully dense acts and for any $i, j \in I$, there exists an epimorphism $\varphi_{ij} : A_i \longrightarrow A_j$, then $A = \coprod_{i\in I} A_i$ is a (cyclically) fully dense act.*

Proof. (i). Let $\{A_i\}_{i\in I}$ be a family of S-acts such that $A = \coprod_{i\in I} A_i$ is a fully dense act. Suppose that $i \in I$, B is a subact of A_i and $a \in A_i$. By assumption, there exists a homomorphism $f : B \longrightarrow A$ such that $f(b) = a$. We have $B = \coprod_{j\in J} B_j$ such that each B_j is indecomposable. So $b \in B_k$ for some $k \in J$. First, we show that $f(B_k) \subseteq A_i$. Let $c \in B_k$. Since $b, c \in B_k$,

$$b = b_1 s_1,\ b_1 t_1 = b_2 s_2,\ ...,\ b_n t_n = c$$

for some $b_i \in B_k, s_i, t_i \in S, i \in \mathbb{N}$. Then

$$f(b) = f(b_1)s_1,\ f(b_1)t_1 = f(b_2)s_2,\ ...,\ f(b_n)t_n = f(c).$$

So $f(b), f(c)$ are in one component. Thus $f(c) \in A_i$, and so $f(B_k) \subseteq A_i$. Now, define $g : B \longrightarrow A_i$ by

$$g(c) = \begin{cases} f(c)\ ; & c \in B_k \\ c & \text{otherwise.} \end{cases} \tag{1}$$

Clearly, g is a homomorphism and $g(b) = a$. Therefore, A_i is a fully dense act.

(ii). Let $\{A_i\}_{i\in I}$ be a family of fully dense acts, and $A = \prod_{i\in I} A_i$. Suppose that B is a subact of A and $a = \{a_i\}_{i\in I} \in A$. Let $j \in I$ and $B_j = \{c \in A_j |\ \exists b = \{b_i\}_{i\in I} \in B, c = b_j\}$. Clearly, B_j is a subact of A_j. So there exists a homomorphism $f_j : B_j \longrightarrow A_j$ such that $f_j(c_j) = a_j$ for some c_j in B_j. Define $f : B \longrightarrow A$ by $f(\{b_i\}_{i\in I}) = \{f_i(b_i)\}_{i\in I}$. It is easily checked that f is a homomorphism and $f(\{c_i\}_{i\in I}) = \{f_i(a_i)\}_{i\in I} = \{a_i\}_{i\in I}$, and we are done.

(iii). Let $a \in A$ and $B = \coprod_{l\in L} B_l$ is a subact of A. If $a \in A_i$ for some $i \in I$ and $B_k = B \cap A_k \neq \emptyset$ for some $k \in L$, then by assumption, $a = g(a_k)$ where $g : A_k \longrightarrow A_i$ is an epimorphism and $a_k \in A_k$. Since A_k is a fully dense act, $a_k = f_k(b_k)$, for some homomorphism $f_k : B_k \longrightarrow A_k$ and some $b_k \in B_k$. Now define the homomorphism $f : B \longrightarrow A$, by

$$f(b) = \begin{cases} (gof_k)(b)\ ; & b \in B_k \\ b\ ; & \text{otherwise} \end{cases} \tag{2}$$

It is clear that $f(b_k) = a$ and so A is a fully dense act. One can prove for cyclically fully dense act similarly. □

It is easy to see that a retract of a (cyclically) fully dense act is a (cyclically) fully dense act. Now using part (iii) of Proposition 2.2, the next corollary holds. Note that an S-act F is a free S-act if and only if $F \cong \coprod_{i\in I} S_i$ where for any $i \in I, S_i \cong S_S$ (see Theorem 1.5.13 of [3]).

Corollary 2.3. *The following statements are equivalent for a monoid S:*

(i) *Every projective S-act is a (cyclically) fully dense act.*

(ii) *Every free S-act is a (cyclically) fully dense act.*

(iii) *S is a (cyclically) fully dense monoid.*

(iv) *Every (principal) right ideal of S is a generator.*

Regarding the concepts of dense subacts and CQ-injective acts, the proof of the following proposition is straightforward.

Proposition 2.4. *Let S be a monoid and $B \subseteq C \subseteq A$ be S-acts. Then the following hold:*

(i) *If $B \subseteq_d A$ and A is CQ-injective, then $C \subseteq_d A$.*

(ii) *If $B \subseteq_d A$ and C is CQ-injective, then $B \subseteq_d C$.*

(iii) *Any CQ-injective subact of any fully dense act is a fully dense act.*

Now, we study conditions under which a cyclic S-act is a cyclically fully dense act. Recall that for every right congruence ρ on S and for every $s \in S$, the right congruence ρ_s is defined by $(x, y) \in \rho_s \Longleftrightarrow (sx, sy) \in \rho$.

Proposition 2.5. *For a right congruence ρ on S, S/ρ is a cyclically fully dense act if and only if for each $s \in S$ there exist $t, u \in S$ such that $\rho s \leqslant \rho t$ and $tu\,\rho\,1$.*

Proof. Necessity. If $s \in S$, then by assumption $S/\rho = \mathrm{Tr}([s]S, S/\rho)$ and consequently $[1] = f([s]u)$ for some $u \in S$ and some homomorphism $f : [s]S \longrightarrow S/\rho$. If $f([s]) = [t]$, then $[1] = f([s]u) = [t]u$, and so $tu\,\rho\,1$. To show that $\rho_s \leqslant \rho_t$, let $x\rho_s y$. So $[s]x = [s]y$. Thus $[t]x = f([s]x) = f([s]y) = [t]y$ which means that $tx\,\rho\,ty$. Therefore, $x\rho_t y$, and we are done.

Sufficiency. Suppose that $s \in S$. By assumption, there exist $t, u \in S$ such that $\rho_s \leqslant \rho_t$ and $tu\,\rho\,1$. Define $f : [s]S \longrightarrow S/\rho$ by $f([s]v) = [t]v$ for each $v \in S$. Since $\rho_s \leqslant \rho_t$, clearly f is a well-defined S-homomorphism. Moreover, from $tu\,\rho\,1$ we conclude that f is an epimorphism, and the result follows. □

3 Prime and strongly prime S-acts

In this section we introduce the notions of prime and strongly prime S-acts and the interrelationship between (strongly) prime acts and fully dense acts is investigated. Recall that an S-act A is called *strongly faithful* if the equality $as = at$ implies that $s = t$ for $s, t \in S$ and $a \in A$. In what follows as an over class of strongly faithful acts, we define the notion of strongly prime acts. Recall that for an element a of an S-act A, the annihilator of a is defined by ann(a):=$\{(s,t) \in S \times S |\ as = at\} = ker(\lambda_a)$ where $\lambda_a : S_S \longrightarrow A$ is defined by $\lambda_a(s) = as$ for every $s \in S$. Also the annihilator of A is defined by ann(A)=$\bigcap_{a \in A}$ ann(a) (see [1]).

Definition 3.1. Let S be a monoid and A be an S-act. Then A is called *strongly prime*, if for any elements $a, b \in A$, ann(a)=ann(b). Also A is said to be *prime*, if for any subact B of A, ann(B) = ann(A).

It is obvious that for a monoid S, an S-act A is prime if and only if for each $a \in A$, ann(aS)=ann(A). Also (strongly) prime S-acts are preserved under product and the right S-act, S_S is strongly prime if and only if S is left cancellative. Evidently, we have the following implications:
(strongly faithful → strongly prime → prime) and also (faithful strongly prime ↔ strongly faithful).

The following example shows that the implications, (strongly faithful → strongly prime → prime) are strict.

Example 3.2. (i)] Suppose S is a non-trivial monoid and $A = \Theta \amalg \Theta$. Then obviously A is strongly prime which is not strongly faithful.
(ii) Suppose $S = T^1$ where T is a right zero semigroup with at least two elements. It is clear that for any $x \in T$, ann(x) =$\{(1, x), (x, 1)\} \cup \Delta_S$. Now

if $x, y \in T$ and $A = xS \cup yS$, then we can easily see that A is a prime S-act which is not strongly prime.

Proposition 3.3. *All S-acts are (strongly) prime if and only if $S = \{1\}$.*

Proof. Since $S \coprod \Theta$ is prime, we have

$$\bigtriangledown_S = S \times S = \mathrm{ann}(\Theta) = \mathrm{ann}(S \coprod \Theta) \subseteq \mathrm{ann}(S) = \Delta_S$$

which implies that $S = \{1\}$. The converse is clear. □

Lemma 3.4. *Suppose A and B are isomorphic strongly prime S-acts over a monoid S. Then for any elements $a \in A, b \in B$, $ann(a) = ann(b)$.*

Proof. Suppose $f : A \longrightarrow B$ is an isomorphism and $a \in A$, $b \in B$. Thus for some $x \in A, f(x) = b$ which implies that $\mathrm{ann}(x) = \mathrm{ann}(b)$. Since A is strongly prime, $\mathrm{ann}(x) = \mathrm{ann}(a)$ and so $\mathrm{ann}(a) = \mathrm{ann}(b)$. □

Recall from [3], that over a monoid S an S-act A satisfies Condition (E), if for all $a \in A, s, t \in S$, $as = at$ implies that there exist $b \in A, z \in S$ such that $a = bz$ and $zs = zt$

Proposition 3.5. *The following statements are equivalent for a monoid S:*

(i) *Every projective S-act is strongly prime.*
(ii) *Every free S-act is strongly prime.*
(iii) *Every S-act which satisfies Condition E is strongly prime.*
(iv) *S_S is strongly prime.*

Proof. (ii) ⇒ (i). Clearly a retract of a strongly prime S-act is strongly prime. Also by Proposition 3.17.4 of [3], every projective S-act is a retract of a free S-act and so the result follows.

(iv) ⇒ (ii). Holds by the above lemma and Theorem 1.5.13 of [3].

(iv) ⇒ (iii). Let an S-act A satisfy Condition (E). If $(s, t) \in \mathrm{ann}(a)$, then $as = at$ and by assumption there exist $u \in S$ and $a' \in A$ such that $a = a'u$, $us = ut$. By assumption, $s = t$ and hence $\mathrm{ann}(a) = \Delta_S$. The other implications are clear. □

By a routine argument, we can see that S_S is a prime S-act if and only if for each $s, u, v \in S$, if for all $t \in S$, $stu = stv$, then $u = v$. Thus by a similar proof of the previous proposition, we have the following proposition.

Proposition 3.6. *The following statements are equivalent for a monoid S:*

(i) *Every projective S-act is prime.*
(ii) *Every free S-act is prime.*
(iii) *S_S is prime.*
(iv) *For each $s, u, v \in S$, if for all $t \in S$, $stu = stv$, then $u = v$.*

Proposition 3.7. *Let S be a non-trivial commutative monoid. Then the following conditions are equivalent:*

(i) *For every S-act A and for any non-zero elements a and b of A, $ann(a){=}ann(b)$.*
(ii) *$S \cong \mathbb{Z}_p$, where $\mathbb{Z}_p$ denotes a cyclic group of prime order.*

Proof. (i) $\Rightarrow$ (ii). Suppose $\lambda \neq S \times S$ is a right congruence on S. If $A = S \cup \frac{S}{\lambda}$, then by assumption, $\Delta_S = \text{ann}(S) = \text{ann}(\frac{S}{\lambda}) = \lambda$. Thus $Con(S) = \{\bigtriangledown_S\ , \Delta_S\}$ and by Theorem 11 of [1], the result follows.

(ii) $\Rightarrow$ (i). By a routine argument, we can see that for a commutative group S, $Con(S) = \{\bigtriangledown_S, \Delta_S\}$. Thus for any non-zero element a of an S-act A, $\text{ann}(a) = \Delta_S$ and the result follows. □

Next we consider (strongly) prime cyclic S-acts.

Proposition 3.8. *Let ρ be a right congruence on S. Then the following hold:*

(i) *The act S/ρ is prime if and only if for each $s \in S$, $(u, v) \in \bigtriangledown_S$ if $(\forall t \in S,\ stu\ \rho\ stv)$, then $(\forall t \in S,\ tu\ \rho\ tv)$.*
(ii) *The act S/ρ is strongly prime if and only if for each $s \in S$ if $su\ \rho\ sv$ for some $(u, v) \in \bigtriangledown_S$, then $tu\ \rho\ tv$ for each $t \in S$.*

Proof. We only prove part (i). The proof of part (ii) is simillar.
Necessity. Let $s \in S$ and $(u, v) \in \bigtriangledown_S$. Suppose that for all $t \in S$, $stu\, \rho\, stv$. So $(u, v) \in \text{ann}([s]t)$ for each $t \in S$. Then $(u, v) \in \text{ann}([s]S)$. Since $\text{ann}([s]S) = \text{ann}(S/\rho)$, $(u, v) \in \text{ann}(S/\rho) = \bigcap_{t\in S} \text{ann}([t])$ which means that $tu\, \rho\, tv$ for each $t \in S$.

Sufficiency. Let $[s] \in S/\rho$, and $(u, v) \in \text{ann}([s]S)$. Since $\text{ann}([s]S) = \bigcap_{t\in S} \text{ann}([s]t)$, $stu\ \rho\ stv$ for all $t \in S$. By assumption, $tu\ \rho\ tv$ for all $t \in S$. So $(u, v) \in \bigcap_{t\in S} \text{ann}([t]) = \text{ann}(S/\rho)$. Therefore, $\text{ann}([s]S) = \text{ann}(S/\rho)$. □

The next proposition connects the notions of prime S-acts and fully dense acts.

Proposition 3.9. *Let S be a monoid, then the following hold:*

(i) *For S-acts A, B if $B \subseteq_d A$, then $ann(B) = ann(A)$.*
(ii) *Every cyclically fully dense act is prime.*
(iii) *Every CQ-injective strongly prime S-act is a fully dense act.*

Proof. (i). It is clear that $\text{ann}(A) \subseteq \text{ann}(B)$. Suppose that $(s, t) \in \text{ann}(B)$ and $a \in A$. By assumption there exist a homomorphism $f : B \longrightarrow A$ and $b \in B$ such that $a = f(b)$. Hence $\text{ann}(B) \subseteq \text{ann}(b) \subseteq \text{ann}(a)$ for each $a \in A$, and then $\text{ann}(B) \subseteq \text{ann}(A)$.

(ii). Clearly an S-act A is prime if for any cyclic subact $aS, \text{ann}(aS) = \text{ann}(A)$. Now by part (i), the result follows. (iii). Considering the fact that, in every strongly prime act, all cyclic subacts are isomorphic, the condition CQ-injectivity implies the result. □

For an S-act A the set of endomorphisms of A is denoted by $\text{End}(A)$ which forms a monoid under composition of mapping (see [3]).

Corollary 3.10. *Suppose S is a commutative monoid and A is a CQ-injective S-act. If $T = End(A)$, then the following conditions are equivalent:*

(i) *A is strongly prime.*
(ii) *A is a fully dense act.*
(iii) *If $a \in A$, then $A = Ta$, where $Ta = \{f(a) \mid f \in T\}$.*
(iv) *A is a simple T-act.*

Proof. (i) $\Leftrightarrow$ (ii). This is clear by Proposition 3.9.

(i)$\Leftrightarrow$ (iii). A same technique to the proof of Proposition 3.9(iii), can be used.

(iii) $\Leftrightarrow$ (iv). Trivial. □

From [4], a subact B of an S-act A is *fully invariant* if for any homomorphism $f : A \longrightarrow A, f(B) \subseteq B$. Also A is called *duo* if any subact of A is fully invariant. Recall that a subact D of an S-act A is called *essential* in A and is denoted by $D \subseteq' A$, if any homomorphism $h : A \longrightarrow C$ such that $h|_D$ is a monomorphism is itself a monomorphism (see [3]).

Lemma 3.11. *Let A be an S-act over a monoid S. Then the following conditions are equivalent:*

(i) *For any $a, b \in A$ there exists $t \in S$ such that $ann(at) \subseteq ann(b)$.*
(ii) *A is contained in any fully invariant subact of $E(A)$.*

Proof. (i) $\Rightarrow$ (ii). Let Q be a fully invariant subact of $E(A)$ and $b \in A$. Since A is an essential subact of $E(A)$, there exists $\theta \neq a \in A \cap Q$. By assumption, for some $t \in S, \mathrm{ann}(at) \subseteq \mathrm{ann}(b)$. Thus $f : atS \longrightarrow bS$ is defined by $f(atx) = bx$ for any $x \in S$ is a well-defined homomorphism which can be extended to an endomorphism of $E(A)$. Now since $at \in Q$ and Q is a fully invariant in $E(A), b = f(at) = \bar{f}(at) \in Q$ and the result follows.

(ii) $\Rightarrow$ (i). Suppose $a, b \in A$. If $Q := \bigcup_{f \in \mathrm{End}(E(A))} f(aS)$, then Q is a fully invariant subact of $E(A)$ and so by assumption $A \subseteq Q$. Thus $b = f(at)$ for some $t \in S$ and consequently $\mathrm{ann}(at) \subseteq \mathrm{ann}(b)$. □

In the next theorem, we present more information concerning prime acts over commutative monoids.

Theorem 3.12. *Suppose S is a commutative monoid and A is an S-act. Then the following statements are equivalent:*

(i) *A is strongly prime.*

(ii) *A is prime.*

(iii) *$ann(A) = ann(a)$ for every element $a \in A$.*

(iv) *A is a cyclically fully dense act.*

(v) *$aS \subseteq_d aS \cup bS$ for any elements $a, b \in A$.*

(vi) *$aS \cong bS$ for any elements $a, b \in A$.*

(vii) *A is contained in any fully invariant subact of $E(A)$.*

Proof. (i) $\Rightarrow$ (ii), (i) $\Rightarrow$ (iii) and (i) $\Rightarrow$ (vi) are clear.

(vi) $\Rightarrow$ (i). Let $f : aS \longrightarrow bS$ be an isomorphism for elements $a, b \in A$. If $f(a) = bt$ for some $t \in S$ then $\mathrm{ann}(a) \subseteq \mathrm{ann}(bt)$. Thus since S is commutative, $\mathrm{ann}(a) \subseteq \mathrm{ann}(bS) = \mathrm{ann}(b)$ and the result follows.

(vi) $\Rightarrow$ (iv). Suppose $a, b \in A$ and $aS \cong bS$, then $b \in \mathrm{Tr}(aS, A)$. Thus $A \subseteq \mathrm{Tr}(aS, A)$.

(iv) $\Rightarrow$ (iii) It follows by part (i) of Proposition 3.9.

(vi) $\Rightarrow$ (v). If $aS \cong bS$, then $bS \subseteq_d \mathrm{Tr}(aS, aS \cup bS) = bS \cup aS$.

(v) $\Rightarrow$ (vi). By assumption and by Proposition 3.9, for any elements $a, b \in A$, $\mathrm{ann}(aS) = \mathrm{ann}(bS)$ and since S is commutative, $\mathrm{ann}(a) = \mathrm{ann}(b)$. Thus clearly $aS \cong bS$.

(i) $\Longleftrightarrow$ (vii). Apply Lemma 2.16. □

From [2] an S-act A is called *Rees artinian* if it satisfies the descending chain condition on its subacts. In particular a monoid S is said to be *right Rees artinian* if the right S-act S_S is Rees artinian.

Since S_S is a projective (cyclic) S-act, by Proposition 2.8 of [5] and by part (iii) of Proposition 2.2, we obtain the following result. Note that over a group S any cyclic S-act is a simple act (fully dense act).

Corollary 3.13. *Let S be a right Rees artinian monoid. Then the following conditions are equivalent:*

(i) *Every cyclic S-act is a fully dense act.*

(ii) *Every projective S-act is a fully dense act.*

(iii) *S is a group.*

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