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A note on hyperideals in ordered hypersemigroups

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Abstract. For an ordered hypersemigroup H, we denote by $\mathcal{N}$the semilattice congruence on H defined by $x\mathcal{N}y$ if and only if the hyperfilters of H generated by the elements x and y coincide. We first prove that this is a complete semilattice congruence on H. Moreover, if H is an ordered hypersemigroup, T a hyperfilter of H and, for a class $(z)_{\mathcal{N}}$ of H there is an element in the intersection $T \cap (z)_{\mathcal{N}}$, then the class $(z)_{\mathcal{N}}$ is a subset of T. From these two statements, the following two important results can be obtained: (1) If H is an ordered hypersemigroup, then each hyperideal of some $(z)_{\mathcal{N}}$-class of H does not contain proper hyperfilters. As a consequence, (2) every prime hyperideal of an ordered hypersemigroup is decomposable into its $\mathcal{N}$-classes.

1. Introduction

The concept of the hypergroup introduced by the French Mathematician F. Marty at the 8th Congress of Scandinavian Mathematicians in 1933 is as follows: A *hypergroup* is a nonempty set H endowed with a multiplication xy such that the following assertions are satisfied: (i) $xy \subseteq H$; (ii) $x(yz) = (xy)z$; (iii) $xH = Hx = H$ for every $x, y, z \in H$ (see [10]). The first researchers who investigate hypergroups using the definition given by Marty were Mittas and Corsini. The concept of the hypersemigroup follows at the usual way and in the recent years many groups in the world investigate these two subjects, related subjects as well, in research programs and hundreds of papers on hypergroups and hypersemigroups appeared using the definition introduced by Marty. As it is no possible to refer to all of them, we will mention only some, related to the present paper, in the references such as the [1–5, 7–10, 12]. If H is a hypergroupoid, a relation σ on H is called *congruence* if $(a, b) \in \sigma$ and $c \in H$ implies $(c \circ a, c \circ b) \in \sigma$ and $(a \circ c, b \circ c) \in \sigma$; in the sense that for every $u \in c \circ a$ and every $v \in c \circ b$ we have $(u, v) \in \sigma$ and for every $u \in a \circ c$ and every $v \in b \circ c$ we have $(u, v) \in \sigma$. A congruence σ on H is called *semilattice congruence* if, for any $a, b \in H$, we have $(a, a \circ a) \in \sigma$ and $(a \circ b, b \circ a) \in \sigma$; in the sense that for every $a \in H$ and every $u \in a \circ a$ we have $(a, u) \in \sigma$ and for every $a, b \in H$, every $u \in a \circ b$ and every $v \in b \circ a$, we have $(u, v) \in \sigma$. An *ordered hypergroupoid* is an ordered set

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$(H, \leq)$ at the same time a hypergroupoid such that $a \leq b$ implies $x \circ a \leq x \circ b$ and $a \circ x \leq b \circ x$ for all $x \in H$, in the sense that for every $x \in H$ and every $u \in x \circ a$ there exists $v \in x \circ b$ such that $u \leq v$ and for every $u \in a \circ x$ there exists $v \in b \circ x$ such that $u \leq v$ [5]. A nonempty subset I of an ordered hypergroupoid is called a *hyperideal* of H if the following hold: (1) $H \circ I \subseteq I$ and $I \circ H \subseteq I$ and (2) if $a \in I$ and $b \in H$ such that $b \leq a$, then $b \in I$. A hyperideal I of H is called *prime* if (1) $a, b \in H$ such that $a \circ b \subseteq I$ implies $a \in I$ or $b \in I$ and (2) for every $a, b \in H$, either $a \circ b \subseteq I$ or $(a \circ b) \cap I = \emptyset$. A hyperideal (hyperfilter) T of H is called *proper* if H is the only hyperideal (hyperfilter) of H. If I is an ideal of an $\mathcal{N}$-class of a semigroup, then I has no completely prime ideals. As a consequence every complete prime ideal of a semigroup is a union of $\mathcal{N}$-classes [11]. In ordered semigroups, we always use the terms prime, weakly prime instead of completely prime, prime given by Petrich and we will keep the same for hypersemigroups as well. For an ordered hypersemigroup H, we denote by $\mathcal{N}$ the semilattice congruence on H defined by $x\mathcal{N}y$ if and only if $N(x) = N(y)$, where $N(a)$ is the hyperfilter of H generated by the element a of H. The present paper deals with the decomposition of prime hyperideals of an ordered hypersemigroup into its $\mathcal{N}$-classes. First of all, the class $\mathcal{N}$ is a complete semilattice congruence on H. If now H is an ordered hypersemigroup, T a hyperfilter of H and z an element of H that belongs to $T \cap (z)_{\mathcal{N}}$, then the class $(z)_{\mathcal{N}}$ is a subset of T, that yields to the following two important results: Firstly, if H is an ordered hypersemigroup, $z \in H$ and I a hyperideal of $(z)_{\mathcal{N}}$, then I does not contain proper hyperfilters, as so does not contain proper prime hyperideals as well. Secondly, if H is an ordered hypersemigroup and I a prime hyperideal of H, then I is decomposable into its $\mathcal{N}$-classes. The corresponding results on hypersemigroup (or semigroups) (without order) can be also obtained as application of the results of the present paper as each hypersemigroup (semigroup) endowed with the equality relation "=" is an ordered hypersemigroup (ordered semigroup).

2. Main results

Definition 2.1. Let $(H, \circ, \leq)$ be an ordered hypergroupoid. A nonempty subset F of H is called a *hyperfilter* of H if the following assertions are satisfied:

1) if $x, y \in F$, then $x \circ y \subseteq F$,
2) if $x, y \in H$ such that $x \circ y \subseteq F$, then $x \in F$ and $y \in F$,
3) for any $x, y \in H$, we have $x \circ y \subseteq F$ or $(x \circ y) \cap F = \emptyset$,
4) if $x \in F$ and $y \in H$ such that $y \geq x$, then $y \in F$.

That is, it is a hypersubgroupoid of H satisfying the relations 2–4.

Definition 2.2. Let H be a hypergroupoid. A nonempty subset T of H is called a *prime subset* of H if the following assertions are satisfied:

1) if $a, b \in H$ such that $a \circ b \subseteq T$, then $a \in T$ or $b \in T$ and

2) for every $a, b \in H$, we have $a \circ b \subseteq T$ or $(a \circ b) \cap T = \emptyset$.

Definition 2.3. Let $(H, \circ, \leq)$ be an ordered hypergroupoid. A semilattice congruence σ on H is called *complete* if $a \leq b$ implies $(a, a \circ b) \in \sigma$, in the sense that if $u \in a \circ b$, then $(a, u) \in \sigma$.

Proposition 2.4. *Let $(H, \circ, \leq)$ be an ordered hypergroupoid. Then the semilattice congruence $\mathcal{N}$ is a complete semilattice congruence on H.*

Proof. Let $a \leq b$. Then $(a, a \circ b) \in \mathcal{N}$. In fact: Let $u \in a \circ b$. Then $(a, u) \in \mathcal{N}$, that is $N(a) = N(u)$. Indeed: Since $N(a) \ni a \leq b$, we have $b \in N(a)$. Since $a, b \in N(a)$, we have $a \circ b \subseteq N(a)$. Since $u \in a \circ b$, we have $u \in N(a)$, then $N(u) \subseteq N(a)$. On the other hand, since $u \in a \circ b$ and $u \in N(u)$, we have $u \in (a \circ b) \cap N(u)$. Since $(a \circ b) \cap N(u) \neq \emptyset$, we have $a \circ b \subseteq N(u)$. Then $a \in N(u)$, and $N(a) \subseteq N(u)$. Hence we obtain $N(u) = N(a)$ and the proof is complete. □

In a similar way as in the Lemma in [6] we can prove the following:

Proposition 2.5. *Let H be an ordered hypergroupoid and F a nonempty subset of H. The following are equivalent:*

(1) *F is a hyperfilter of H.*

(2) *$H \backslash F = \emptyset$ or $H \backslash F$ is a prime hyperideal of H.*

Proposition 2.6. *An ordered hypergroupoid H does not contain proper hyperfilters if and only if H does not contain proper prime hyperideals.*

Proof. ($\Rightarrow$). Let I be a prime hyperideal of H and $I \subset H$. Then $\emptyset \neq H \backslash I \subseteq H$ and $H \backslash (H \backslash I) (= I)$ is a prime hyperideal of H ($H \backslash I$ is the complement of I to H). By Proposition 2.5, $H \backslash I$ is a hyperfilter of H. Then $H \backslash I = H$ and $I = \emptyset$ which is impossible.

($\Leftarrow$). Let F be a hyperfilter of H and $F \subset H$. Since $H \backslash F \neq \emptyset$, by Proposition 2.5, $H \backslash F$ is a prime hyperideal of H. Then $H \backslash F = H$ and $F = \emptyset$ which is impossible. □

Remark 2.7. *Let H be an ordered hypergroupoid, T a hyperfilter of H, $z \in H$ and $a \in T \cap (z)_{\mathcal{N}}$. Then we have $(z)_{\mathcal{N}} \subseteq T$.*

Proof. Let $y \in (z)_{\mathcal{N}}$. Then $(y)_{\mathcal{N}} = (z)_{\mathcal{N}} = (a)_{\mathcal{N}}$, so $(y, a) \in \mathcal{N}$ and $N(y) = N(a)$. Since T is a hyperfilter of H containing the element a, we have $N(a) \subseteq T$. Thus we have $y \in N(y) = N(a) \subseteq T$ and so $y \in T$. □

Theorem 2.8. *Let H be an ordered hypersemigroup and $z \in H$. If I is a hyperideal of $(z)_{\mathcal{N}}$, then I does not contain proper hyperfilters.*

Proof. Let F be a hyperfilter of I. Then $F = I$. In fact: Take an element $a \in F$ (such an element exists as F is a nonempty set) and consider the set

$$T := \{x \in H \mid a^2 \circ x \subseteq F\}.$$

Then the following assertions are satisfied:

(1). $F = T \cap I$. Indeed: Let $y \in F$. Since $a^2 \subseteq F$, we have $a^2 \circ y \subseteq F$ and so $y \in T$. Besides, $F \subseteq I$, so $F \subseteq T \cap I$. Let now $y \in T \cap I$. Since $y \in T$, we have $a^2 \circ y \subseteq F$. Then, since $a^2 \subseteq F \subseteq I$, we have $y \in I$ and, since F hyperfilter of I, we have $y \in F$.

(2). T is a hyperfilter of H. In fact: This is a nonempty subset of H because $a^3 \subseteq F$ and $a \in T$. Let $x, y \in T$. Then $x \circ y \subseteq T$. In fact: The following properties are satisfied:

(A). $y \circ a^2 \subseteq F$. Indeed: Since $a^2 \circ y \subseteq F$ and $a^2 \subseteq F$, we have

$$F \supseteq (a^2 \circ y) \circ a^2 = a^2 \circ (y \circ a^2), \text{ where } a^2 \subseteq I \qquad (*)$$

Moreover, we have $y \circ a^2 \subseteq I$. Indeed: Since $a^2 \circ y \subseteq F \subseteq I \subseteq (z)_{\mathcal{N}}$, we have

$$\begin{aligned}(z)_{\mathcal{N}} = (a^2 \circ y)_{\mathcal{N}} &:= (a^2)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (y)_{\mathcal{N}} \circ (a)_{\mathcal{N}} = (y \circ a)_{\mathcal{N}},\end{aligned}$$

so $y \circ a \subseteq (z)_{\mathcal{N}}$. Then, since $a \in F \subseteq I$ and I is a hyperideal of $(z)_{\mathcal{N}}$, we have

$$y \circ a^2 = (y \circ a) \circ a \subseteq (z)_{\mathcal{N}} \circ I \subseteq I,$$

thus $y \circ a^2 \subseteq I$. Since $a^2 \subseteq I$ and $y \circ a^2 \subseteq I$, by $(*)$, we have $y \circ a^2 \subseteq F$.

(B). $a^2 \circ x \circ y \subseteq I$. In fact: Clearly, $a^2 \circ x \circ y = a \circ (a \circ x \circ y)$. On the other hand, $a \circ x \circ y \subseteq (z)_{\mathcal{N}}$. Indeed: Since $a^2 \circ x \subseteq F \subseteq I \subseteq (z)_{\mathcal{N}}$, we have

$$(z)_{\mathcal{N}} = (a^2 \circ x)_{\mathcal{N}} = (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} = (a)_{\mathcal{N}} \circ (x)_{\mathcal{N}} = (a \circ x)_{\mathcal{N}}.$$

We have seen in (A) that $(z)_{\mathcal{N}} = (y \circ a)_{\mathcal{N}}$ $(= (a \circ y)_{\mathcal{N}})$. Thus we have

$$\begin{aligned}(z)_{\mathcal{N}} = (z^2)_{\mathcal{N}} &:= (z)_{\mathcal{N}} \circ (z)_{\mathcal{N}} = (a \circ x)_{\mathcal{N}} \circ (a \circ y)_{\mathcal{N}} \\ &= (a)_{\mathcal{N}} \circ \Big((x)_{\mathcal{N}} \circ (a)_{\mathcal{N}}\Big) \circ (y)_{\mathcal{N}} = (a)_{\mathcal{N}} \circ \Big((a)_{\mathcal{N}} \circ (x)_{\mathcal{N}}\Big) \circ (y)_{\mathcal{N}} \\ &= (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a)_{\mathcal{N}} \circ (x \circ y)_{\mathcal{N}} = (a \circ x \circ y)_{\mathcal{N}}\end{aligned}$$

and so $a \circ x \circ y \subseteq (z)_{\mathcal{N}}$. Then, since I is a hyperideal of $(z)_{\mathcal{N}}$, we have

$$a \circ (a \circ x \circ y) \subseteq I \circ (z)_{\mathcal{N}} \subseteq I,$$

and so $a^2 \circ x \circ y \subseteq I$.

Since $x \in T$, we have $a^2 \circ x \subseteq F$. Then, by (A), $(a^2 \circ x) \circ (y \circ a^2) \subseteq F$, and then we have

$$F \supseteq (a^2 \circ x) \circ (y \circ a^2) = (a^2 \circ x \circ y) \circ a^2,$$

where $a^2 \subseteq I$ and $a^2 \circ x \circ y \subseteq I$ (by (B)). Since F is a hyperfilter of I, we have $a^2 \circ x \circ y \subseteq F$ and so $x \circ y \subseteq T$.

If $x, y \in H$ such that $x \circ y \subseteq H$, then $x \in T$ and $y \in T$. In fact, since $x \circ y \subseteq T$, we have $a^2 \circ x \circ y \subseteq F$. We remark first that

$$F \supseteq (a^2 \circ x \circ y) \circ a^2 = (a^2 \circ x) \circ (y \circ a^2) \qquad (*)$$

In addition, the following properties are satisfied:

(A). $a^2 \circ x \subseteq I$. In fact: We have $a^2 \circ x = a \circ (a \circ x)$, where $a \in I$. Moreover, $a \circ x \subseteq (z)_{\mathcal{N}}$. Indeed, since $a^2 \circ x \circ y \subseteq F \subseteq (z)_{\mathcal{N}}$, we have $(a^2 \circ x \circ y) = (z)_{\mathcal{N}}$. Since $a \in F \subseteq I \subseteq (z)_{\mathcal{N}}$, we have $(z)_{\mathcal{N}} = (a)_{\mathcal{N}}$. Thus we get $(a^2 \circ x \circ y)_{\mathcal{N}} = (a)_{\mathcal{N}}$. On the other hand,

$$\begin{aligned}(a \circ x)_{\mathcal{N}} &= (a)_{\mathcal{N}} \circ (x)_{\mathcal{N}} = (a^2 \circ x \circ y)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \\ &= (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \circ (x)_{\mathcal{N}} = (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (a^2)_{\mathcal{N}} \circ (x^2)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (a^2 \circ x \circ y)_{\mathcal{N}} = (z)_{\mathcal{N}},\end{aligned}$$

thus $a \circ x \subseteq (z)_{\mathcal{N}}$. Since I is a hyperideal of $(z)_{\mathcal{N}}$, we have $a \circ (a \circ x) \subseteq I \circ (z)_{\mathcal{N}}$ and so $a^2 \circ x \subseteq I$.

(B). $y \circ a^2 \subseteq I$. In fact: First of all, $y \circ a^2 = (y \circ a) \circ a$ and $a \in I$. Besides, $y \circ a \subseteq (a)_{\mathcal{N}}$, Indeed, since

$$\begin{aligned}(y \circ a)_{\mathcal{N}} &= (y)_{\mathcal{N}} \circ (a)_{\mathcal{N}} = (y)_{\mathcal{N}} \circ (a^2 \circ x \circ y)_{\mathcal{N}} = (a^2 \circ x \circ y)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (a^2 \circ x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a^2 \circ x)_{\mathcal{N}} \circ (y^2)_{\mathcal{N}} \\ &= (a^2 \circ x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a^2 \circ x \circ y)_{\mathcal{N}} = (z)_{\mathcal{N}},\end{aligned}$$

$y \circ a$ is a subset of $(z)_{\mathcal{N}}$. Since $a \in I$ and I a hyperideal of $(z)_{\mathcal{N}}$, we get $(y \circ a) \circ a \subseteq (z)_{\mathcal{N}} \circ I \subseteq I$ and so $y \circ a^2 \subseteq I$.

Since $a^2 \circ x \subseteq I$, $y \circ a^2 \subseteq I$ and F is a hyperfilter of I, by $(*)$, we have $a^2 \circ x \subseteq F$ and $y \circ a^2 \subseteq F$. Finally, $y \circ a^2 \subseteq F$ implies $a^2 \circ y \subseteq F$. In fact: Since $a^2, y \circ a^2 \subseteq F$, we have $a^2 \circ (y \circ a^2) \subseteq F$, then $(a^2 \circ y) \circ a^2 \subseteq F$. On the other hand, $a^2 \circ y \subseteq I$. Indeed: Since $a \circ y \subseteq (a \circ y)_{\mathcal{N}} = (z)_{\mathcal{N}}$ and $a \in I$ $(F \subseteq I)$, we have

$$a^2 \circ y = a \circ (a \circ y) \subseteq I \circ (z)_{\mathcal{N}} \subseteq I,$$

so $a^2 \circ y \subseteq I$. Since $(a^2 \circ y) \circ a^2 \subseteq F$, $a^2 \circ y \subseteq I$, $a^2 \subseteq I$ and F is a hyperfilter of I, we have $a^2 \circ y \subseteq F$.

For any $x, y \in T$ it is clear that either $a \circ b \subseteq T$ or $(a \circ b) \cap T = \emptyset$. Finally, let $x \in T$ and $y \in H$ such that $y \geq x$. Then $y \in T$. In fact: We have $a^2 \circ y \geq a^2 \circ x$ and $a^2 \circ x \subseteq F$. It is enough to prove that $a^2 \circ y \subseteq I$. Then, since F is a hyperfilter of I, we have $a^2 \circ y \subseteq F$ and so $y \in T$. On the other site, $a^2 \circ y = a \circ (a \circ y)$, where $a \in F \subseteq I \subseteq (z)_{\mathcal{N}}$. We prove that $a \circ y \subseteq (z)_{\mathcal{N}}$. Then, since I is a hyperideal of $(z)_{\mathcal{N}}$, we have $a \circ (a \circ y) \subseteq I \circ (z)_{\mathcal{N}}$ and so $a^2 \circ y \subseteq I$. First of all, since $a^2 \circ x \subseteq F \subseteq I \subseteq (z)_{\mathcal{N}}$, we have

$$(z)_{\mathcal{N}} = (a^2 x)_{\mathcal{N}} := (a^2)_{\mathcal{N}} (x)_{\mathcal{N}} = (a)_{\mathcal{N}} (x)_{\mathcal{N}} = (a \circ x)_{\mathcal{N}}.$$

On the other hand, since $x \le y$, we have $a \circ x \le a \circ y$ then, by Proposition 2.4, we have $(a \circ x, a \circ x \circ a \circ y) \in \mathcal{N}$, hence we obtain

$$\begin{aligned}(a \circ x)_{\mathcal{N}} &= (a \circ x \circ a \circ y)_{\mathcal{N}} := (a)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (a)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (a^2)_{\mathcal{N}} \circ (x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (a^2 \circ x)_{\mathcal{N}} \circ (y)_{\mathcal{N}} = (z)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \\ &= (a)_{\mathcal{N}} \circ (y)_{\mathcal{N}} \text{ (since } a \in (z)_{\mathcal{N}}) \\ &= (a \circ y)_{\mathcal{N}}.\end{aligned}$$

Hence $a \circ y \subseteq (a \circ y)_{\mathcal{N}} = (a \circ x)_{\mathcal{N}} = (z)_{\mathcal{N}}$. Since T is a hyperfilter of H, $a \in T$ and $a \in (z)_{\mathcal{N}}$, by Remark 2.7, we have $(z)_{\mathcal{N}} \subseteq T$. Thus we have

$$I \subseteq F = T \cap I \supseteq (z)_{\mathcal{N}} \cap I = I,$$

and then $F = I$. □

By Proposition 2.5 and Theorem 2.8 we have the following

Corollary 2.9. *If H is an ordered hypersemigroup, $z \in H$ and I a hyperideal of $(z)_{\mathcal{N}}$, then I does not contain proper prime hyperideals (of I).*

Theorem 2.10. *Let H be an ordered hypersemigroup and I a prime hyperideal of H. Then we have $I = \bigcup\limits_{x \in I} \{(x)_{\mathcal{N}} \mid x \in I\}$.*

Proof. Let $t \in (x)_{\mathcal{N}}$ for some $x \in I$. Since $(x)_{\mathcal{N}}$ is a hyperideal of (the hypersemigroup) $(x)_{\mathcal{N}}$, by Corollary 2.9, $(x)_{\mathcal{N}}$ does not contain proper prime hyperideals. We prove that $(x)_{\mathcal{N}} \cap I$ is a prime hyperideal of $(x)_{\mathcal{N}}$. Then we get $(x)_{\mathcal{N}} \cap I = (x)_{\mathcal{N}}$, and then $t \in I$. First of all, $(x)_{\mathcal{N}} \cap I$ is a nonempty subset of $(x)_{\mathcal{N}}$ and this is because $x \in (x)_{\mathcal{N}}$ and $x \in I$. Moreover we have

$$\begin{aligned}(x)_{\mathcal{N}} \circ ((x)_{\mathcal{N}} \cap I) &\subseteq (x)^2_{\mathcal{N}} \cap (x)_{\mathcal{N}} \circ I = (x^2)_{\mathcal{N}} \cap (x)_{\mathcal{N}} \circ I \\ &= (x)_{\mathcal{N}} \cap (x)_{\mathcal{N}} \circ I \subseteq (x)_{\mathcal{N}} \cap H \circ I \\ &\subseteq (x)_{\mathcal{N}} \cap I\end{aligned}$$

and $((x)_{\mathcal{N}} \cap I) \circ (x)_{\mathcal{N}} \subseteq (x)^2_{\mathcal{N}} \cap I \circ (x)_{\mathcal{N}} \subseteq (x)_{\mathcal{N}} \cap I \circ H \subseteq (x)_{\mathcal{N}} \cap I$. In addition, if $a \in (x)_{\mathcal{N}} \cap I$ and $b \in (x)_{\mathcal{N}}$ such that $b \le a$ then, since $b \le a \in I$ and I is a hyperideal of H, we have $b \in I$. Thus we have $b \in (x)_{\mathcal{N}} \cap I$.

Let now $y, z \in (x)_{\mathcal{N}}$ such that $y \circ z \subseteq (x)_{\mathcal{N}} \cap I$. Since $y \circ z \subseteq I$ and I is a prime hyperideal of H, we have $y \in I$ or $z \in I$. Hence $y \in (x)_{\mathcal{N}} \cap I$ or $z \in (x)_{\mathcal{N}} \cap I$ and the proof is complete. □

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