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Semi-prime and meet weak closure operations in lower BCK-semilattices

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Abstract. The notion of semi-prime (resp., meet) weak closure operation is introduced, and related properties are investigated. Characterizations of a semi-prime (resp. meet) closure operation are discussed. Examples which show that the notion of semi-prime weak closure operation is independent to the notion of meet weak closure operation.

1. Introduction

Semi-prime closure operations on ideals of BCK-algebras are introduced in the paper [2], and a finite type of closure operations on ideals of BCK-algebras are discussed in [1]. As a general form of closure operations on ideals of BCK-algebras, Bordbar et al. [3] introduced the notion of weak closure operations on ideals of BCK-algebras. Regarding weak closure operation, they defined finite type and (strong) quasi-primeness, and investigated related properties. They also discussed positive implicative (resp., commutative and implicative) weak closure operations, and provided several examples to illustrate notions and properties.

In this paper, we introduce the notion of semi-prime (resp., meet) weak closure operation in lower BCK-semilattices, and investigate their properties. We discuss characterizations of a semi-prime (resp. meet) closure operation. We provide examples to show that the notion of semi-prime weak closure operation is independent to the notion of meet weak closure operation.

2. Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers. We refer the reader to the books [5, 6] for further information regarding BCK/BCI-algebras.

Suppose that X is a BCK-algebra. Define a binary relation $\leqslant$ on X as follows:

$$x \leqslant y \text{ if and only if } x * y = 0$$

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for any $x, y \in X$. Then $(X, \leqslant)$ is a partially ordered set (see [5]), and we say that $\leqslant$ is the BCK-ordering on X.

A partially ordered set $(X, \leqslant)$ is called a *lower* (resp., *upper*) *semilattice* if any two elements in X have the greatest lower bound (resp., least upper bound). If $(X, \leqslant)$ is both a lower semilattice and an upper semilattice, we call it a *lattice* (see [5]).

A BCK-algebra X is called a *lower BCK-semilattice* (see [6]) if X is a lower semilattice with respect to the BCK-order.

In what follows, let X be a lower BCK-semilattice and $\mathcal{I}(X)$ a set of all ideals of X unless otherwise specified.

Definition 1 ([3])**.** An element x of X is called a *zeromeet element* of X if the condition

$$(\exists\, y \in X \setminus \{0\})\,(x \wedge y = 0)$$

is valid. Otherwise, x is called a *non-zeromeet element* of X.

For a subset A of a BCK-algebra X, denote by $\langle A \rangle$ the generated ideal by A. If $A = \{a\}$, then $\langle A \rangle$ is denoted by $\langle a \rangle$.

Denote by $Z(X)$ the set of all zeromeet elements of X, that is,

$$Z(X) = \{x \in X \mid x \wedge y = 0 \text{ for some nonzero element } y \in X\}.$$

Definition 2 ([4])**.** For any nonempty subsets A and B of X, we define a set

$$(A :_{\wedge} B) := \{x \in X \mid x \wedge B \subseteq A\}$$

which is called the *relative annihilator* of B with respect to A.

Lemma 1 ([4])**.** *If A and B are ideals of X, then the relative annihilator $(A :_{\wedge} B)$ of B with respect to A is an ideal of X.*

Definition 3 ([3])**.** A mapping $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ is called a *weak closure operation* on $\mathcal{I}(X)$ if the following conditions are valid.

$$(\forall A \in \mathcal{I}(X))\,(A \subseteq cl(A))\,, \tag{1}$$

$$(\forall A, B \in \mathcal{I}(X))\,(A \subseteq B \;\Rightarrow\; cl(A) \subseteq cl(B))\,. \tag{2}$$

In what follows, we use A^{cl} instead of $cl(A)$.

3. Semi-prime and meet weak closure operations

Definition 4. For any nonempty subsets A and B of X, we denote

$$A \wedge B := \langle \{a \wedge b \mid a \in A, b \in B\} \rangle$$

which is called the *meet ideal* of X generated by A and B. In this case, we say that the operation "$\wedge$" is a *meet operation.* If $A = \{a\}$, then $\{a\} \wedge B$ is denoted by $a \wedge B$. Also, if $B = \{b\}$, then $A \wedge \{b\}$ is denoted by $A \wedge b$.

Theorem 1. *If A and B are ideals of X, then so is the meet set*

$$A \wedge B = \{a \wedge b \mid a \in A,\ b \in B\}$$

based on A and B.

Proof. Obviously, $0 \in A \wedge B$. Let $x \in A \wedge B$ and $y * x \in A \wedge B$ for $x, y \in X$. Then $x = a \wedge b$ and $y * x = a' \wedge b'$ where $a, a' \in A$ and $b, b' \in B$. Since $a \wedge b \leqslant a$ and A is an ideal, we have $x = a \wedge b \in A$. Similarly, we have

$$y * x = a' \wedge b' \leqslant a' \in A.$$

Since A is an ideal of X, it follows that $y \in A$. By the similar way, we get $y \in B$. Therefore,

$$y = y \wedge y \in \{a \wedge b \mid a \in A,\ b \in B\} = A \wedge B$$

and $A \wedge B$ is an ideal of X. □

Obviously, $A \wedge B = B \wedge A$ for any nonempty subsets A and B of X. If A and B are ideals of X, then

$$A \wedge B = \{a \wedge b \mid a \in A, b \in B\}.$$

Given ideals A and B of X, we consider two ideals

$$A \wedge B^{cl} \text{ and } (A \wedge B)^{cl},$$

and investigate their relations where "cl" is a weak closure operation on $\mathcal{I}(X)$.

The following example shows that there exist ideals A and B of X such that

$$A \wedge B^{cl} \nsubseteq (A \wedge B)^{cl}.$$

Example 1. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	3	0

Note that X has five ideals: $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 2\}, A_3 = \{0, 1, 2\}$ and $A_4 = X$. Define a map $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ by $A_0^{cl} = A_0$, $A_1^{cl} = A_3$, $A_2^{cl} = A_3$, $A_3^{cl} = A_4$ and $A_4^{cl} = A_4$. Then "cl" is a weak closure operation on $\mathcal{I}(X)$, and

$$A_1 \wedge A_2^{cl} = A_1 \wedge A_3^{cl} = A_1 \nsubseteq A_0 = A_0^{cl} = (A_1 \wedge A_2)^{cl}.$$

Proposition 1. *For any element a of X, we have*

$$\langle a\rangle = a \wedge X. \tag{3}$$

Proof. Suppose that $p \in a \wedge X$. Then there exist $b_1, b_2, \ldots, b_n \in \{a \wedge x \mid x \in X\}$ such that $(\ldots((p * b_1) * b_2) * \ldots) * b_n = 0$. Let $b_i = a \wedge a_i$ where $a_i \in X$ for $i = 1, 2, \ldots, n$. Since $b_1 \leqslant a$, it follows from (a2) that

$$(p * a) * b_2 \leqslant (p * b_1) * b_2. \tag{4}$$

Since $b_2 \leqslant a$, we have

$$(p * a) * a \leqslant (p * a) * b_2. \tag{5}$$

By (4) and (5), we have

$$(p * a) * a \leqslant (p * b_1) * b_2.$$

Continuing this way, we get

$$p * a^n = (\ldots((p * a) * a) * \ldots) * a \leqslant (\ldots((p * b_1) * b_2) * \ldots) * b_n = 0.$$

Hence $p * a^n = 0$, that is, $p \in \langle a\rangle$. Therefore $a \wedge X \subseteq \langle a\rangle$.

Conversely, suppose that $p \in \langle a\rangle$. Then there exists $n \in \mathbb{N}$ such that $p * a^n = 0$, that is, $(\ldots((p * a) * a) * \ldots) * a = 0$. Since $a = a \wedge a$, we conclude that

$$(\ldots((p * (a \wedge a)) * (a \wedge a)) * \ldots) * (a \wedge a) = 0.$$

Also $a \wedge a \in \{a \wedge x \mid x \in X\}$, and so $p \in \langle\{a \wedge x \mid x \in X\}\rangle = a \wedge X$. Therefore $a \wedge X = \langle a\rangle$. □

Definition 5. A weak closure operation "*cl*" on $\mathcal{I}(X)$ is said to be *semi-prime* if

$$(\forall A, B \in \mathcal{I}(X))(A \wedge B^{cl} \subseteq (A \wedge B)^{cl}). \tag{6}$$

Example 2. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	4	0

Note that X has five ideals $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 1, 2\}$, $A_3 = \{0, 1, 2, 3\}$ and $A_4 = X$. Define a map $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ by $A_0^{cl} = A_0$, $A_1^{cl} = A_2$, $A_2^{cl} = A_2$, $A_3^{cl} = A_4$ and $A_4^{cl} = A_4$. It is routine to check that "*cl*" is a semi-prime weak closure operation on $\mathcal{I}(X)$.

Proposition 2. *If "cl" is a semi-prime weak closure operation on $\mathcal{I}(X)$, then*

$$(\forall a \in X)(\forall A \in \mathcal{I}(X))\left(a \wedge A^{cl} \subseteq (a \wedge A)^{cl}\right). \tag{7}$$

Proof. Suppose that "*cl*" is a semi-prime weak closure operation on $\mathcal{I}(X)$. Then

$$a \wedge A^{cl} \subseteq \langle a \rangle \wedge A^{cl} \subseteq (\langle a \rangle \wedge A)^{cl} = (a \wedge A)^{cl}$$

for any $a \in X$ and $A \in \mathcal{I}(X)$ by using Proposition 1. □

In Proposition 2, if "*cl*" is a weak closure operation on $\mathcal{I}(X)$ which is not semi-prime, then (7) is not true in general, that is, there exist $a \in X$ and $A \in \mathcal{I}(X)$ such that $a \wedge A^{cl} \nsubseteq (a \wedge A)^{cl}$ as seen in the following example.

Example 3. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

Then $Z(X) = \{0\}$ and X has nine ideals: $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 1, 3\}$, $A_3 = \{0, 1, 2\}$, $A_4 = \{0, 1, 4\}$, $A_5 = \{0, 1, 2, 3\}$, $A_6 = \{0, 1, 3, 4\}$, $A_7 = \{0, 1, 2, 4\}$ and $A_8 = X$. Let $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ be a function defined by $A_4^{cl} = A_6$, $A_7^{cl} = A_8$ and $A_i^{cl} = A_i$ for $i = 1, 2, 3, 5, 6, 8$. Then "*cl*" is a weak closure operation on $\mathcal{I}(X)$. But it is not semi-prime since

$$A_4^{cl} \wedge A_2 = A_6 \wedge A_2 = A_2 \nsubseteq A_1 = A_1^{cl} = (A_4 \wedge A_2)^{cl}.$$

On the other hand, we have

$$3 \wedge A_4^{cl} = 3 \wedge A_6 = A_2 \nsubseteq A_1 = A_1^{cl} = (3 \wedge A_4)^{cl}$$

for a non-zeromeet element 3 of X.

We provide conditions for a weak closure operation to be semi-prime.

Theorem 2. *If a weak closure operation "cl" on $\mathcal{I}(X)$ satisfies the condition* (7), *then it is semi-prime.*

Proof. We first show that

$$(\langle b_1 \rangle + \langle b_2 \rangle) \wedge A \subseteq \langle b_1 \rangle \wedge A + \langle b_2 \rangle \wedge A \tag{8}$$

for all $A \in \mathcal{I}(X)$ and $b_1, b_2 \in X$. If $z \in (\langle b_1\rangle + \langle b_2\rangle) \wedge A$, then there exist $x \in \langle b_1\rangle + \langle b_2\rangle$ and $a \in A$ such that $z = x \wedge a$. Since $x \in \langle b_1\rangle + \langle b_2\rangle = \langle\langle b_1\rangle \cup \langle b_2\rangle\rangle$, we have

$$(\ldots((x * s_1) * s_2) * \ldots) * s_n = 0 \tag{9}$$

for some $s_i \in \langle b_1\rangle \cup \langle b_2\rangle$, $1 \leqslant i \leqslant n$. Since $s_i \in \langle b_1\rangle$ or $s_i \in \langle b_2\rangle$ for $i \in \{1, 2, \ldots, n\}$, it follows from (9) that $x \in \langle b_1\rangle$ or $x \in \langle b_2\rangle$. Hence

$$z = x \wedge a \in \langle b_1\rangle \wedge A \text{ or } z = x \wedge a \in \langle b_2\rangle \wedge A,$$

and thus $z \in \langle b_1\rangle \wedge A + \langle b_2\rangle \wedge A$. Therefore (8) is valid. Let B be an ideal of X. Then $B = \sum\limits_{b\in B} \langle b\rangle$ and $\langle b\rangle \wedge A^{cl} = b \wedge X \wedge A^{cl} = b \wedge A^{cl}$ by Proposition 1. It follows from (8) and (7) that

$$B \wedge A^{cl} = \left(\sum_{b\in B}\langle b\rangle\right) \wedge A^{cl} \subseteq \sum_{b\in B}\left(\langle b\rangle \wedge A^{cl}\right) = \sum_{b\in B}\left(b \wedge A^{cl}\right) \subseteq \sum_{b\in B}(b \wedge A)^{cl}.$$

Since $b \in B = \sum\limits_{b\in B}\langle b\rangle$, we have $b \wedge A \subseteq \sum\limits_{b\in B}\langle b\rangle \wedge A$ and so

$$(b \wedge A)^{cl} \subseteq \left(\sum_{b\in B}\langle b\rangle \wedge A\right)^{cl}.$$

Hence $\sum\limits_{b\in B}(b \wedge A)^{cl} \subseteq \left(\sum\limits_{b\in B}\langle b\rangle \wedge A\right)^{cl} = (B \wedge A)^{cl}$. Therefore $B \wedge A^{cl} \subseteq (B \wedge A)^{cl}$ and "cl" is a semi-prime weak closure operation on $\mathcal{I}(X)$. $\square$

Definition 6. A weak closure operation "cl" on $\mathcal{I}(X)$ is said to be *meet* if it satisfies:

$$(\forall A \in \mathcal{I}(X))\,(\forall a \in X \setminus Z(X))\,\left((a \wedge A)^{cl} = a \wedge A^{cl}\right). \tag{10}$$

Example 4. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	2	1	0	0
4	4	4	4	4	0

Note that X has five ideals $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 2\}$, $A_3 = \{0, 1, 2, 3\}$ and $A_4 = X$. Let $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ be a function defined by $A_0^{cl} = A_0$, $A_1^{cl} = A_3$, $A_2^{cl} = A_3$, $A_3^{cl} = A_3$ and $A_4^{cl} = A_4$. By routine calculations, we know that "cl" is a meet weak closure operation on $\mathcal{I}(X)$.

The following example shows that there exists a weak closure operation that is not meet.

Example 5. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	4	4	4	0

Recall that X has six ideals: $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 2\}$, $A_3 = \{0, 1, 2, 3\}$, $A_4 = \{0, 1, 4\}$ and $A_5 = X$. Let $cl : \mathcal{I}(X) \to \mathcal{I}(X)$ be a function defined by $A_0^{cl} = A_0$, $A_1^{cl} = A_3$, $A_2^{cl} = A_4$, $A_3^{cl} = A_3$, $A_4^{cl} = A_4$ and $A_5^{cl} = A_5$. Then "cl" is a weak closure operation on $\mathcal{I}(X)$, but it is not meet since

$$3 \wedge A_1^{cl} = 3 \wedge A_3 = A_1 \neq A_3 = A_1^{cl} = (3 \wedge A_1)^{cl}$$

for a non-zeromeet element 3 of X.

We consider relations between $(z \wedge A :_\wedge z)$ and A for any ideal A and $z \in X \setminus Z(X)$. We can easily prove that $A \subseteq (z \wedge A :_\wedge z)$. But the reverse inclusion is not true in general as seen in the following example.

Example 6. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

For a non-zeromeet element 2 and an ideal $A = \{0, 1, 2\}$ of X, we have

$$(2 \wedge \{0, 1, 2\} :_\wedge 2) = X \nsubseteq A.$$

In the following proposition, we discuss conditions for the inclusion $(z \wedge A :_\wedge z) \subseteq A$ to be true. We first consider the following condition:

$$(\forall a, b \in X)(\forall z \in X \setminus Z(X))((a * b) \wedge z \leq (a \wedge z) * (b \wedge z)), \tag{11}$$

The following examples show that the inequality (11) does not hold in general.

Example 7. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	1
3	3	3	3	0

Note that 1 is a non-zeromeet element of X and

$$(2 * 3) \wedge 1 = 1 \not\leqslant 0 = (2 \wedge 1) * (3 \wedge 1),$$

which shows that the inequality (11) is not true.

Proposition 3. *If X satisfies the condition* (11)*, then $(z \wedge A :_\wedge z) \subseteq A$ and hence $(z \wedge A :_\wedge z) = A$ for every $A \in \mathcal{I}(X)$ and $z \in X \setminus Z(X)$.*

Proof. Suppose that $a \in (z \wedge A :_\wedge z)$. Then $a \wedge z \in z \wedge A$, and so there exist $a_1, a_2, \ldots, a_n \in A$ such that

$$(\ldots((a \wedge z) * (a_1 \wedge z)) * (a_2 \wedge z)) * \ldots * (a_n \wedge z) = 0.$$

It follows from the condition (11) that

$$\begin{aligned}&(\ldots((a * a_1) * a_2) * \ldots * a_n) \wedge z\\ &\leqslant (\ldots((a \wedge z) * (a_1 \wedge z)) * (a_2 \wedge z)) * \ldots * (a_n \wedge z) = 0\end{aligned}$$

and so that $(\ldots((a * a_1) * a_2) * \ldots * a_n) \wedge z = 0$. Since $z \in X \setminus Z(X)$, it follows that

$$(\ldots((a * a_1) * a_2) * \ldots) * a_n = 0.$$

Hence $a \in A$, and therefore $(z \wedge A :_\wedge z) \subseteq A$. □

The following example illustrates Proposition 3.

Example 8. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	4	4	0

Note that 4 is the only non-zeromeet element of X and there are nine ideals: $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 2\}$, $A_3 = \{0, 3\}$, $A_4 = \{0, 1, 2\}$, $A_5 = \{0, 1, 3\}$, $A_6 = \{0, 2, 3\}$, $A_7 = \{0, 1, 2, 3\}$ and $A_8 = X$. We know that

$$(4 \wedge A_i :_\wedge 4) = (A_i :_\wedge 4) = A_i$$

for $i = 0, 1, 2, \ldots, 8$.

We consider a characterization of a meet weak closure operation.

Theorem 3. *Let X satisfy the condition* (11) *and let "cl" be a weak closure operation on $\mathcal{I}(X)$. Then "cl" is meet if and only if it satisfies the following properties:*

$$\langle a\rangle^{cl} = \langle a\rangle \text{ and } A^{cl} = ((a \wedge A)^{cl} :_{\wedge} a) \tag{12}$$

for any $a \in X \setminus Z(X)$ and any ideal A of X.

Proof. Suppose that "cl" is a meet weak closure operation on $\mathcal{I}(X)$. Let a be a non-zeromeet element and A be an ideal of X. Then, by Propositions 1 and 3, we have

$$\langle a\rangle^{cl} = (a \wedge X)^{cl} = a \wedge X^{cl} = a \wedge X = \langle a\rangle$$

and

$$((a \wedge A)^{cl} :_{\wedge} a) = (a \wedge A^{cl} :_{\wedge} a) = A^{cl},$$

respectively.

Conversely, suppose that the condition (12) is valid. For a non-zeromeet element a and an ideal A of X, we have

$$a \wedge A^{cl} = a \wedge ((a \wedge A)^{cl} :_{\wedge} a) \subseteq (a \wedge A)^{cl}.$$

If $z \in (a \wedge A)^{cl}$, then $z \in \langle a\rangle^{cl} = \langle a\rangle$ since $a \wedge A \subseteq \langle a\rangle$. Thus

$$z \in \langle a\rangle^{cl} = \langle a\rangle = a \wedge X,$$

and so $z = a \wedge b$ for some $b \in X$. Hence $a \wedge b \in (a \wedge A)^{cl}$, i.e., $b \in ((a \wedge A)^{cl} :_{\wedge} a) = A^{cl}$. Therefore $z = a \wedge b \in a \wedge A^{cl}$ and the proof is complete. $\square$

The notion of semi-prime weak closure operation is independent to the notion of meet weak closure operation as seen in the following examples.

Example 9. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	1	0
2	2	1	0	2	0
3	3	3	3	0	0
4	4	4	4	4	0

Note that $Z(X) = \{0, 1, 2, 3\}$ and X has five ideals: $A_0 = \{0\}$, $A_1 = \{0, 1, 2\}$, $A_2 = \{0, 3\}$, $A_3 = \{0, 1, 2, 3\}$ and $A_4 = X$. Let "$cl : \mathcal{I}(X) \to \mathcal{I}(X)$" be a mapping defined by $A_0^{cl} = A_1$, $A_1^{cl} = A_3$, $A_2^{cl} = A_3$, $A_3^{cl} = A_3$ and $A_4^{cl} = A_4$. Then "cl" is a meet weak closure operation on $\mathcal{I}(X)$. But it not semi-prime since

$$A_1^{cl} \wedge A_2 = A_3 \wedge A_2 = A_2 \nsubseteq A_1 = A_0^{cl} = (A_1 \wedge A_2)^{cl}.$$

Example 10. Consider a lower BCK-semilattice $X = \{0, 1, 2, 3, 4\}$ with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	0	2
3	3	3	2	0	3
4	4	4	4	4	0

Then $Z(X) = \{0\}$ and X has five ideals: $A_0 = \{0\}$, $A_1 = \{0, 1\}$, $A_2 = \{0, 1, 2, 3\}$, $A_3 = \{0, 1, 4\}$ and $A_4 = X$. Let "$cl : \mathcal{I}(X) \to \mathcal{I}(X)$" be a mapping defined by $A_0^{cl} = A_1$, $A_1^{cl} = A_4$, $A_2^{cl} = A_4$, $A_3^{cl} = A_4$ and $A_4^{cl} = A_4$. Then "cl" is a semi-prime weak closure operation on $\mathcal{I}(X)$. But it is not meet since

$$4 \wedge A_2^{cl} = 4 \wedge A_4 = A_3 \neq A_4 = (4 \wedge A_2)^{cl}.$$

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