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Filter theory in EQ-algebras based on soft sets

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Abstract. Int-soft prefilters (filters) of EQ-algebras are introduced, and related properties are investigated. Characterizations of int-soft prefilters (filters) of EQ-algebras are provided.

1. Introduction

Many-valued logics are uniquely determined by the algebraic properties of the structure of its truth values. As a precise logic to deal with uncertainty and approximate reasoning, one can consider fuzzy logics. As well-known fuzzy logics, one can also take residuated lattices based on fuzzy logics such as Łukasiewicz logic, BL-logic, R_0-logic, MTL-logic, and so forth. In fuzzy logics, it is generally accepted that the algebraic structure should be a residuated lattice. MV-algebras, BL-algebras, R_0-algebras, MTL-algebras, and so forth are well-known classes of residuated lattices. A new class of algebras called EQ-algebras has been recently introduced by V. Novák and B. De Baets [9] with the intent to develop an algebraic structure of truth values for fuzzy type theory. From the point of view of logic, the main difference between residuated lattices and EQ-algebras lies in the way the implication operation is obtained. It is obtained from a (strong) conjuction in residuated lattices, but it is obtained from equivalence in EQ-algebras. Consequently, the two types of algebras differ in several essential points, despite their many similar or identical properties. An EQ-algebra has three binary operations: meet ($\wedge$), multiplication ($\otimes$), and fuzzy equality ($\sim$), and a unit element, whereas the implication ($\rightarrow$) is derived from the fuzzy equality ($\sim$). Filter theory plays a vital role in studying several algebraic structures such as residuated lattices, MV-algebras, BL-algebras, R_0-algebras, MTL-algebras, BCK/BCI-algebras, lattice implication algebras, and so forth. M. El-Zekey et al. [2] have introduced and studied the prefilters and filters of EQ-algebras. Liu and Zhang [5] have introduced and studied the implicative and positive implicative prefilters (filters) of EQ-algebras.

Soft set theory [8] has been firstly proposed by a Russian researcher Molodtsov in 1999. This is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. Generally, the soft set theory is different from traditional tools for dealing with uncertainties, such as the theory of probability, the theory

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of fuzzy sets and the theory of rough sets. Nowadays, work on the soft set theory is progressing rapidly. Maji et al. [7] has been firstly defined some operations on soft sets. They also have been introduced the soft set into the decision-making problem [6] that is based on the concept of knowledge reduction in the rough set theory [10]. Jun et al. [4] has been introduced and studied int-soft filters, int-soft G-filters, regular int-soft filters, and MV-int-soft filters in residuated lattices. Jun et al. has been studied (implicative) int-soft filters of R_0-algebras (see [3]).

The aim of this paper is to study prefilters (filters) and positive implicative prefilters (filters) of EQ-algebras based on soft set theory. We study characterizations of positive implicative int-soft prefilters (filters) of EQ-algebras, and establish the extension property for positive implicative int-soft filters.

2. Preliminaries

We display basic definitions and properties of EQ-algebras that will be used in this paper. For more details of EQ-algebras, we refer the reader to [1], [2], and [5].

By an *EQ-algebra* we mean an algebra $E := (E, \wedge, \otimes, \sim, 1)$ of type $(2, 2, 2, 0)$ in which the following axioms are valid:

(E1) $(E, \wedge, 1)$ is a commutative idempotent monoid (i.e., $\wedge$-semilattice with top element 1),

(E2) $(E, \otimes, 1)$ is a monoid and $\otimes$ is isotone with respect to $\leqslant$ (with $x \leqslant y$ defined as $x \wedge y = x$),

(E3) $x \sim x = 1$,

(E4) $((x \wedge y) \sim z) \otimes (a \sim x) \leqslant z \sim (a \wedge y)$,

(E5) $(x \sim y) \otimes (a \sim b) \leqslant (x \sim a) \sim (y \sim b)$,

(E6) $(x \wedge y \wedge z) \sim x \leqslant (x \wedge y) \sim x$,

(E7) $x \otimes y \leqslant x \sim y$

for all $x, y, z, a, b \in E$.

The operation "$\wedge$" is called *meet* (*infimum*) and "$\otimes$" is called *multiplication*. If the multiplication is commutative in an EQ-algebra E, then we say that E is a commutative EQ-algebra.

Let E be an EQ-algebra. For all $x \in L$, we put $\tilde{x} = x \sim 1$. We also define the derived operation, so called *implication* and denoted by $\to$, as follows:

$$(\forall x, y \in E)\,(x \to y = (x \wedge y) \sim x)\,. \tag{1}$$

An EQ-algebra E is said to be *residuated* if $(x \otimes y) \wedge z = x \otimes y$ if and only if $x \wedge ((y \wedge z) \sim y) = x$ for all $x, y, z \in E$.

Proposition 2.1. *Every (commutative) EQ-algebra E satisfies the following conditions for all $a, b, c, d \in E$:*

(1) *If* $a \leqslant b$, *then* $a \to b = 1$, $a \sim b = b \to a$, $\tilde{a} \leqslant \tilde{b}$, $c \to a \leqslant c \to b$ *and* $b \to c \leqslant a \to c$,

(2) $a \otimes b \leqslant a \wedge b \leqslant a, b$ *and* $b \otimes a \leqslant a \wedge b \leqslant a, b$,

(3) $a \to b = a \to (a \wedge b)$,

(4) $(a \to b) \otimes (b \to c) \leqslant a \to c$,

(5) $a \to b \leqslant (a \wedge c) \to (b \wedge c)$.

A subset F of an EQ-algebra E is called a *prefilter* of E if it satisfies the following conditions:

$$1 \in F, \tag{2}$$

$$(\forall a, b \in E)\,(a \to b \in F,\ a \in F \ \Rightarrow\ b \in F)\,. \tag{3}$$

A subset F of an EQ-algebra E is called a *filter* of E if it is a prefilter of E with the following additional condition:

$$(\forall a, b, c \in E)\,(a \to b \in F \ \Rightarrow\ (a \otimes c) \to (b \otimes c) \in F,\ (c \otimes a) \to (c \otimes b) \in F)\,. \tag{4}$$

A prefilter (resp. filter) F of an EQ-algebra E is said to be *positive implicative* if the following assertion is valid:

$$(\forall x, y, z \in E)\,(x \to (y \to z) \in F,\ x \to y \in F \ \Rightarrow\ x \to z \in F)\,. \tag{5}$$

A soft set theory is introduced by Molodtsov [8]. In what follows, let U be an initial universe set and X be a set of parameters. Let $P(U)$ denotes the power set of U and $A, B, C, \ldots \subseteq X$.

A *soft set* $(\tilde{f}, A)$ of X over U is defined to be the set of ordered pairs

$$(\tilde{f}, A) := \{(x, \tilde{f}(x)) : x \in X,\ \tilde{f}(x) \in P(U)\},$$

where $\tilde{f} : X \to P(U)$ such that $\tilde{f}(x) = \emptyset$ if $x \notin A$.

3. Int-soft prefilters (filters)

In what follows, let E denote a commutative EQ-algebra unless otherwise specified.

Definition 3.1. A soft set $(\tilde{f}, E)$ on E over U is called an *int-soft prefilter* (resp. *int-soft filter*) of E if the set

$$i_E(\tilde{f}; \gamma) := \{x \in E \mid \gamma \subseteq \tilde{f}(x)\}$$

is a prefilter (resp. filter) of E for all $\gamma \in P(U)$ with $i_E(\tilde{f}; \gamma) \neq \emptyset$.

We say that $i_E(\tilde{f}; \gamma)$ is the γ-*inclusive set* of $(\tilde{f}, E)$.

Example 3.2. Let $E = \{0, a, b, 1\}$ be a chain. We define two binary operations '$\otimes$' and '$\sim$' by the following tables:

$\otimes$	0	a	b	1
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
1	0	a	b	1

$\sim$	0	a	b	1
0	1	a	a	a
a	a	1	b	b
b	a	b	1	1
1	a	b	1	1

Then $E := (E, \wedge, \otimes, \sim, 1)$ is an EQ-algebra (see [5]). The derived operation "$\rightarrow$" is described as the following table:

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	a	b	1	1
1	a	b	1	1

Then a soft set $(\tilde{f}, E)$ on E over $U = \mathbb{Z}$ defined by

$$\tilde{f}(x) := \begin{cases} 4\mathbb{N} & \text{if } x \in \{0, a\}, \\ 4\mathbb{Z} & \text{if } x = b, \\ 2\mathbb{Z} & \text{if } x = 1 \end{cases}$$

is an int-soft prefilter of E.

Example 3.3. Let E by as in the previous example an let

$\otimes$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
1	0	a	b	1

$\sim$	0	a	b	1
0	1	0	0	0
a	0	1	a	a
b	0	a	1	1
1	0	a	1	1

Then $E := (E, \wedge, \otimes, \sim, 1)$ is an EQ-algebra (see [5]). The derived operation "$\rightarrow$" is described by table:

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	a	1	1
1	0	a	1	1

Then a soft set $(\tilde{f}, E)$ on E over $U = \mathbb{Z}$ defined as follows:

$$\tilde{f}(x) := \begin{cases} 4\mathbb{Z} & \text{if } x \in \{0, a\}, \\ 2\mathbb{Z} & \text{if } x \in \{b, 1\} \end{cases}$$

is an int-soft filter of E.

Theorem 3.4. *A soft set* $(\tilde{f}, E)$ *on* E *over* U *is an int-soft prefilter of* E *if and only if the following assertions are valid.*

$$(\forall x \in E)\left(\tilde{f}(x) \subseteq \tilde{f}(1)\right), \tag{6}$$

$$(\forall x, y \in E)\left(\tilde{f}(x) \cap \tilde{f}(x \to y) \subseteq \tilde{f}(y)\right). \tag{7}$$

Proof. Assume that $(\tilde{f}, E)$ is an int-soft prefilter of E. For any $x \in E$, let $\tilde{f}(x) = \gamma$. Then $x \in i_E(\tilde{f}; \gamma)$, and so $i_E(\tilde{f}; \gamma) \neq \emptyset$. Thus $i_E(\tilde{f}; \gamma)$ is a prefilter of E, and therefore $1 \in i_E(\tilde{f}; \gamma)$. Hence $\tilde{f}(1) \supseteq \gamma = \tilde{f}(x)$ for all $x \in E$. For any $x, y \in E$, let $\tilde{f}(x) \cap \tilde{f}(x \to y) = \delta$. Then $\tilde{f}(x) \supseteq \delta$ and $\tilde{f}(x \to y) \supseteq \delta$, that is, $x \in i_E(\tilde{f}; \delta)$ and $x \to y \in i_E(\tilde{f}; \delta)$. It follows from (3) that $y \in i_E(\tilde{f}; \delta)$ and that $\tilde{f}(y) \supseteq \delta = \tilde{f}(x) \cap \tilde{f}(x \to y)$.

Conversely, let $(\tilde{f}, E)$ be a soft set on E over U that satisfies two conditions (6) and (7). Let $\varepsilon \in P(U)$ be such that $i_E(\tilde{f}; \varepsilon) \neq \emptyset$. Then $\tilde{f}(a) \supseteq \varepsilon$ for some $a \in i_E(\tilde{f}; \varepsilon)$. Using (6), we have $\tilde{f}(1) \supseteq \tilde{f}(a) \supseteq \varepsilon$, and so $1 \in i_E(\tilde{f}; \varepsilon)$. Let $x, y \in E$ be such that $x \in i_E(\tilde{f}; \varepsilon)$ and $x \to y \in i_E(\tilde{f}; \varepsilon)$. Then $\varepsilon \subseteq \tilde{f}(x)$ and $\varepsilon \subseteq \tilde{f}(x \to y)$. It follows from (7) that $\varepsilon \subseteq \tilde{f}(x) \cap \tilde{f}(x \to y) \subseteq \tilde{f}(y)$ and that $y \in i_E(\tilde{f}; \varepsilon)$. Hence $i_E(\tilde{f}; \varepsilon)$ is a prefilter of E for all $\varepsilon \in P(U)$ with $i_E(\tilde{f}; \varepsilon) \neq \emptyset$, and therefore $(\tilde{f}, E)$ is an int-soft prefilter of E. □

Theorem 3.5. *A soft set* $(\tilde{f}, E)$ *on* E *over* U *is an int-soft filter of* E *if and only if it satisfies* (6), (7) *and*

$$(\forall x, y, z \in E)\left(\tilde{f}(x \to y) \subseteq \tilde{f}((x \otimes z) \to (y \otimes z))\right). \tag{8}$$

Proof. Let $(\tilde{f}, E)$ be an int-soft filter of E. Then $(\tilde{f}, E)$ is an int-soft prefilter of E, and so two conditions (6) and (7) are valid by Theorem 3.4. Let $x, y \in E$ and $\tau \in P(U)$ be such that $\tilde{f}(x \to y) = \tau$. Then $x \to y \in i_E(\tilde{f}; \tau)$. Since $i_E(\tilde{f}; \tau)$ is a filter of E, we have $(x \otimes z) \to (y \otimes z) \in i_E(\tilde{f}; \tau)$ for all $x, y, z \in E$. It follows that

$$\tilde{f}((x \otimes z) \to (y \otimes z)) \supseteq \tau = \tilde{f}(x \to y)$$

for all $x, y, z \in E$.

Conversely, let $(\tilde{f}, E)$ be a soft set on E over U that satisfies (6), (7) and (8). Then $(\tilde{f}, E)$ is an int-soft prefilter of E by Theorem 3.4, and thus $i_E(\tilde{f}; \gamma)$ is a prefilter of E for all $\gamma \in P(U)$ with $i_E(\tilde{f}; \gamma) \neq \emptyset$. Let $x, y \in E$ be such that $x \to y \in i_E(\tilde{f}; \gamma)$. Then

$$\tilde{f}((x \otimes z) \to (y \otimes z)) \supseteq \tilde{f}(x \to y) \supseteq \gamma$$

by (8), and so $(x \otimes z) \to (y \otimes z) \in i_E(\tilde{f}; \gamma)$. Hence $i_E(\tilde{f}; \gamma)$ is a filter of E, and therefore $(\tilde{f}, E)$ is an int-soft filter of E. □

Proposition 3.6. *Every int-soft prefilter* $(\tilde{f}, E)$ *of* E *for all* $x, y \in E$ *satisfies the following assertions:*

(1) *if* $x \leqslant y$, *then* $\tilde{f}(x) \subseteq \tilde{f}(y)$,

(2) $\tilde{f}(x \otimes y) \subseteq \tilde{f}(x) \cap \tilde{f}(y)$.

Proof. (1). Let $x, y \in E$ be such that $x \leqslant y$. Then $x \to y = 1$ by Proposition 2.1. It follows from (6) and (7) that $\tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(x \to y) = \tilde{f}(x) \cap \tilde{f}(1) = \tilde{f}(x)$.

(2). Using Proposition 2.1(2) and item (1), we have $\tilde{f}(x \otimes y) \subseteq \tilde{f}(x) \cap \tilde{f}(y)$. □

Theorem 3.7. *For a soft set* $(\tilde{f}, E)$ *on* E *over* U, *the following are equivalent.*

(1) $(\tilde{f}, E)$ *is an int-soft prefilter of* E.

(2) $(\forall x, y, z \in E)\left(x \leqslant y \to z \;\Rightarrow\; \tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(z)\right)$.

(3) $(\forall x, y, z \in E)\left(x \to (y \to z) = 1 \;\Rightarrow\; \tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(z)\right)$.

Proof. (1) ⇒ (2). Let $x, y, z \in E$ be such that $x \leqslant y \to z$. Then $\tilde{f}(x) \subseteq \tilde{f}(y \to z)$ by Proposition 3.6(1). Using (7), we get

$$\tilde{f}(z) \supseteq \tilde{f}(y) \cap \tilde{f}(y \to z) \supseteq \tilde{f}(x) \cap \tilde{f}(y).$$

(2) ⇒ (3). Let $x, y, z \in E$ be such that $x \to (y \to z) = 1$. Then

$$x \leqslant 1 = x \to (y \to z),$$

and so $\tilde{f}(x) \subseteq \tilde{f}(y \to z)$ by (2). Since $y \to z \leqslant y \to z$, it follows from (2) that

$$\tilde{f}(z) \supseteq \tilde{f}(y \to z) \cap \tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(y).$$

(3) ⇒ (1). Since $x \to (x \to 1) = 1$ for all $x \in E$, it follows from (3) that $\tilde{f}(x) \subseteq \tilde{f}(1)$ for all $x \in E$. Note that $(x \to y) \to (x \to y) = 1$ for all $x, y \in E$. Thus $\tilde{f}(x) \cap \tilde{f}(x \to y) \subseteq \tilde{f}(y)$ for all $x, y \in E$ by (3). Therefore $(\tilde{f}, E)$ is an int-soft prefilter of E by Theorem 3.4. □

Proposition 3.8. *For any int-soft filter* $(\tilde{f}, E)$ *of* E, *for all* $x, y, z \in E$ *the following assertions are valid.*

(1) $\tilde{f}(x \otimes y) = \tilde{f}(x) \cap \tilde{f}(y)$,

(2) $\tilde{f}(x \to z) \supseteq \tilde{f}(x \to y) \cap \tilde{f}(y \to z)$.

Proof. (1). The inclusion $\tilde{f}(x \otimes y) \subseteq \tilde{f}(x) \cap \tilde{f}(y)$ follows from Proposition 3.6(2). Note that $y \leqslant 1 \to y$ for all $y \in E$. It follows from Proposition 3.6(1) and (8) that

$$\tilde{f}(y) \subseteq \tilde{f}(1 \to y) \subseteq \tilde{f}((x \otimes 1) \to (x \otimes y)) = \tilde{f}(x \to (x \otimes y))$$

and from (7) that $\tilde{f}(x \otimes y) \supseteq \tilde{f}(x) \cap \tilde{f}(x \to (x \otimes y)) \supseteq \tilde{f}(x) \cap \tilde{f}(y)$ for all $x, y \in E$.

(2). Combining Proposition 2.1(4), Proposition 3.6(1) and item (1) induces

$$\tilde{f}(x \to z) \supseteq \tilde{f}((x \to y) \otimes (y \to z)) = \tilde{f}(x \to y) \cap \tilde{f}(y \to z)$$

for all $x, y, z \in E$. □

4. Int-soft prefilters (filters)

Definition 4.1. A soft set $(\tilde{f}, E)$ on E over U is called a *positive implicative int-soft prefilter (filter)* of E if the nonempty γ-inclusive set of $(\tilde{f}, E)$ is a positive implicative prefilter (filter) of E for all $\gamma \in P(U)$.

Example 4.2. The int-soft filter $(\tilde{f}, E)$ in Example 3.3 is positive implicative, but the int-soft prefilter $(\tilde{f}, E)$ in Example 3.2 is not positive implicative because if we take $\tau \in P(U)$ with $4\mathbb{N} \subsetneq \tau \subseteq 4\mathbb{Z}$, then $i_E(\tilde{f}; \tau) = \{b, 1\}$ is not a positive implicative prefilter of E.

Theorem 4.3. *A soft set $(\tilde{f}, E)$ on E over U is a positive implicative int-soft prefilter (filter) of E if and only if it is an int-soft prefilter (filter) of E that satisfies an additional condition:*

$$(\forall x, y, z \in E) \left(\tilde{f}(x \to (y \to z)) \cap \tilde{f}(x \to y) \subseteq \tilde{f}(x \to z) \right). \tag{9}$$

Proof. Assume that $(\tilde{f}, E)$ is a positive implicative int-soft prefilter (filter) of E. Then $i_E(\tilde{f}; \tau)$ is a positive implicative prefilter (filter) of E for all $\tau \in P(U)$ with $i_E(\tilde{f}; \tau) \neq \emptyset$, and therefore $i_E(\tilde{f}; \tau)$ is a prefilter (filter) of E. Hence $(\tilde{f}, E)$ is an int-soft prefilter (filter) of E. Let $x, y, z \in E$ be such that $\tilde{f}(x \to (y \to z)) \cap \tilde{f}(x \to y) = \varepsilon$. Then $x \to (y \to z) \in i_E(\tilde{f}; \varepsilon)$ and $x \to y \in i_E(\tilde{f}; \varepsilon)$, which implies from (5) that $x \to z \in i_E(\tilde{f}; \varepsilon)$. Thus

$$\tilde{f}(x \to z) \supseteq \varepsilon = \tilde{f}(x \to (y \to z)) \cap \tilde{f}(x \to y).$$

Conversely, let $(\tilde{f}, E)$ be an int-soft prefilter (filter) of E that satisfies (9). Then $i_E(\tilde{f}; \varepsilon)$ is a prefilter (filter) of E for all $\varepsilon \in P(U)$ with $i_E(\tilde{f}; \varepsilon) \neq \emptyset$. Let $x, y, z \in E$ be such that $x \to (y \to z) \in i_E(\tilde{f}; \varepsilon)$ and $x \to y \in i_E(\tilde{f}; \varepsilon)$. Then $\varepsilon \subseteq \tilde{f}(x \to (y \to z))$ and $\varepsilon \subseteq \tilde{f}(x \to y)$. It follows from (9) that

$$\varepsilon \subseteq \tilde{f}(x \to (y \to z)) \cap \tilde{f}(x \to y) \subseteq \tilde{f}(x \to z)$$

and that $x \to z \in i_E(\tilde{f}; \varepsilon)$. Hence $i_E(\tilde{f}; \varepsilon)$ is a positive implicative prefilter (filter) of E for all $\varepsilon \in P(U)$ with $i_E(\tilde{f}; \varepsilon) \neq \emptyset$, and therefore $(\tilde{f}, E)$ is a positive implicative int-soft prefilter (filter) of E. □

Theorem 4.4. *If an int-soft filter of E satisfies the following assertion*

$$(\forall x, y \in E) \left(\tilde{f}(((x \to y) \wedge x) \to y) = \tilde{f}(1) \right), \tag{10}$$

then it is a positive implicative int-soft filter of E.

Proof. Let $(\tilde{f}, E)$ be an int-soft filter of E that satisfies the condition (10). Using Proposition 2.1(5) and Proposition 2.1(3), we have

$$x \to (y \to z) \leqslant (x \wedge y) \to ((y \to z) \wedge y) \quad \text{and} \quad x \to y = x \to (x \wedge y).$$

It follows from (6), Proposition 3.6, Proposition 3.8(2) and (10) that

$$\begin{aligned}&\tilde{f}(x \to y) \cap \tilde{f}(x \to (y \to z)) = \tilde{f}(x \to y) \cap \tilde{f}(x \to (y \to z)) \cap \tilde{f}(1)\\ &\subseteq \tilde{f}(x \to (x \wedge y)) \cap \tilde{f}((x \wedge y) \to ((y \to z) \wedge y)) \cap \tilde{f}(1)\\ &\subseteq \tilde{f}(x \to ((y \to z) \wedge y)) \cap \tilde{f}(((y \to z) \wedge y) \to z) \subseteq \tilde{f}(x \to z).\end{aligned}$$

Therefore $(\tilde{f}, E)$ is a positive implicative int-soft filter of E by Theorem 4.3. □

Theorem 4.5. *Let $(\tilde{f}, E)$ and $(\tilde{g}, E)$ be int-soft filters of E such that $\tilde{f}(1) = \tilde{g}(1)$ and $\tilde{f}(x) \subseteq \tilde{g}(x)$ for all $x \in E$. If $(\tilde{f}, E)$ is positive implicative, then so is $(\tilde{g}, E)$.*

Proof. Indeed, $\tilde{g}(((x \to y) \wedge x) \to y) \supseteq \tilde{f}(((x \to y) \wedge x) \to y) = \tilde{f}(1) = \tilde{g}(1)$, and thus $\tilde{g}(((x \to y) \wedge x) \to y)) = \tilde{g}(1)$ for all $x, y \in E$. Therefore $(\tilde{g}, E)$ is a positive implicative int-soft filter of E by Theorem 4.4. □

References

[1] **M. El-Zekey**, *Representable good EQ-algebras*, Soft Comp. **14** (2010), 1011–1023.

[2] **M. El-Zekey, V. Novák and R. Mesiar**, *On good EQ-algebras*, Fuzzy Sets and Systems **178** (2011), 1–23.

[3] **Y.B. Jun, S.S. Ahn and K.J. Lee**, *Implicative int-soft filters of R_0-algebras*, Discrete Dyn. Nat. Soc. Volume 2013, Article ID 983507, 7 pages.

[4] **Y.B. Jun, S.S. Ahn and K.J. Lee**, *Classes of int-soft filters in residuated lattices*, The Scientific Worlds J., vol. 2014, Article ID 595160, 12 pages.

[5] **L.Z. Liu and X.Y. Zhang**, *Implicative and positive implicative prefilters of EQ-algebras*, J. Intell. Fuzzy Systems **26** (2014), 2087–2097.

[6] **P.K. Maji and A.R. Roy**, *An application of soft sets in a decision making problem*, Comput. Math. Appl. **44** (2002), 1077–1083.

[7] **P.K. Maji, R. Biswas and A R. Roy**, *Soft set theory*, Comput. Math. Appl. **45** (2003), 555–562.

[8] **D. Molodtsov**, *Soft set theory*, Comput. Math. Appl. **37** (1999), 19–31.

[9] **V. Novák and B. De Baets**, *EQ-algebras*, Fuzzy Sets and Systems **160** (2009), 2956–2978.

[10] **Z. Pawlak**, *Rough Sets. Theoretical Aspects of Reasoning about Data*, Kluwer, 1991.

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