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On fuzzy relations and fuzzy quotient Γ-groups

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Abstract

The problem of the structure of fuzzy quotient Γ-groups is discussed. We introduce and define the fuzzy quotient Γ-group by using some special fuzzy relation defined in this paper, and also we prove some basic properties.

1. Introduction and preliminaries

The concept of fuzzy sets was first introduced by Zadeh in [10] and since then there has been a tremendous interest in the subject due to its various applications ranging from engineering and computer since to social behavior studies. The concept of fuzzy relations on a set was defined by Zadeh [10, 11]. The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld [5]. The notion of Γ-groups was introduced in [7] as a generalization of the notion of classical groups. In this paper we introduce and define some new special fuzzy equivalence relations. Then using these relations we define suitable fuzzy quotient Γ-subgroup of G_α/H_α and prove some basic properties.

In 1986 Sen and Saha [7] defined a Γ-semigroup as follows:

Definition 1.1. Let $M = \{a, b, c, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. If there exists a mapping $M \times \Gamma \times M \rightarrow M$ denoted by $(a, \gamma, b) \longmapsto a\gamma b$ and satisfying the identity

$$(a\alpha b)\beta c = a\alpha(b\beta c),$$

where $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-*semigroup*.

For a Γ-semigroup M and a fixed element $\gamma \in \Gamma$ we define on M a binary operation $\circ$ by putting $a \circ b = a\gamma b$ for all $a, b \in M$. Such defined

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groupoid $(M, \circ)$ is denoted by M_γ. It is a semigroup [7]. Moreover, if it is a group for some $\gamma \in \Gamma$, then it is a group for every $\gamma \in \Gamma$ [7]. In this case we says that M is a Γ-*group*. Examples can be found in [7] and [8].

For subsets A and B of a Γ-semigroup M we define the set

$$A\Gamma B = \{a\gamma b \mid a \in A, b \in B \text{ and } \gamma \in \Gamma\}.$$

The interval $[0, 1]$ we denoted by I, $max\{x, y\}$ by $x \vee y$, $min\{x, y\}$ by $x \wedge y$. By a fuzzy set on X we mean any mapping $\mu : X \longmapsto I$. For any fuzzy sets μ and ν on X we define

$$\begin{aligned} \mu = \nu \Leftrightarrow \mu(x) &= \nu(x),\ \forall x \in X, \\ \mu \subseteq \nu \Leftrightarrow \mu(x) &\leqslant \nu(x)\ \forall x \in X, \\ (\mu \cup \nu)(x) &= \mu(x) \vee \nu(x), \\ (\mu \cap \nu)(x) &= \mu(x) \wedge \nu(x). \end{aligned}$$

For a family of fuzzy sets $\{\mu_i \mid i \in I\}$ defined on X we put

$$(\cup\mu_i)(x) = \bigvee_{i\in I}\{\mu_i(x)\} \quad \text{and} \quad (\cap\mu_i)(x) = \bigwedge_{i\in I}\{\mu_i(x)\}.$$

Definition 1.2. A fuzzy set μ of a group G is called a *fuzzy subgroup* if

(i) $\mu(xy) \geqslant \mu(x) \wedge \mu(y)$,

(ii) $\mu(x^{-1}) \geqslant \mu(x)$

holds for all $x, y \in G$.

Obviously $\mu(e) \geqslant \mu(x)$ for every $x \in G$, where e is the identity of G.

Theorem 1.3. *A fuzzy set μ of a group G is a fuzzy subgroup G if and only if*

$$\mu(xy^{-1}) \geqslant \mu(x) \wedge \mu(y) \quad \textit{and} \quad \mu(e) \geqslant \mu(x)$$

for all $x, y \in G$.

Definition 1.4. A fuzzy subgroup μ of a group G is called a *fuzzy normal subgroup* of G if

$$\mu(xyx^{-1}) \geqslant \mu(y)$$

for all $x, y \in G$, or equivalently, if and only if

$$\mu(xy) = \mu(yx)$$

for all $x, y \in G$.

By a *fuzzy relation* on X we mean a fuzzy set $\mu : X \times X \to I$. If θ and φ are two fuzzy relations on a set X, then $\theta \leqslant \varphi$ means that $\theta(x,y) \leqslant \varphi(x,y)$ for all $x, y \in X$. Their composition $\theta \circ \varphi$ is defined by

$$(\theta \circ \varphi)(x,y) = \bigvee_{z \in X} \{\theta(x,z) \wedge \varphi(z,y)\}.$$

Definition 1.5. A fuzzy relation θ on X is a *fuzzy equivalence relation* if

(i) $\theta(x,x) = 1 \ \ \forall x \in X$,

(ii) $\theta(x,y) = \theta(y,x) \ \ \forall x, y \in X$,

(iii) $\theta \circ \theta \leqslant \theta$.

Definition 1.6. A fuzzy equivalence relation θ on a semigroup S is a *fuzzy congruence* if it is *fuzzy compatible*, that is,

$$\theta(x,y) \wedge \theta(z,t) \leqslant \theta(xz, yt)$$

for all $x, y, z, t \in S$, or equivalently, if and only if it is *fuzzy left* and *fuzzy right compatible*, i.e.,

$$\theta(x,y) \leqslant \theta(zx, zy) \quad \text{and} \quad \theta(x,y) \leqslant \theta(xz, yz)$$

for all $x, y, z, t \in S$.

2. Fuzzy relations and fuzzy congruences

We need to define a special relation β_α as follows:

Definition 2.1. Let M be a Γ-group, μ_{H_α} be a fuzzy subgroup of M_α, $\alpha \in \Gamma$ and e_α be the identity of M_α. A fuzzy relation β_α on M is defined by

$$\beta_\alpha(a,b) = \begin{cases} \mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(b), & \text{if } a \neq b, \\ \mu_{H_\alpha}(e_\alpha), & \text{if } a = b. \end{cases}$$

Proposition 2.2. *β_α is a fuzzy equivalence relation on M.*

Proof. β_α is reflexive and symmetric. It is also transitive. Indeed, for all $a, c \in M$ we have

$$\begin{aligned}(\beta_\alpha \circ \beta_\alpha)(a,c) &= \bigvee_{b\in M} \{\beta_\alpha(a,b) \wedge \beta_\alpha(b,c)\} \\ &= \bigvee_{b\in M} \{(\mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(b)) \wedge (\mu_{H_\alpha}(b) \wedge \mu_{H_\alpha}(c))\} \\ &\leqslant \bigvee_{b\in M} \{\mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(b)\} \wedge \bigvee_{b\in M} \{\mu_{H_\alpha}(b) \wedge \mu_{H_\alpha}(c)\} \\ &\leqslant \bigvee_{b\in M} \{\mu_{H_\alpha}(a)\} \wedge \bigvee_{b\in M} \{\mu_{H_\alpha}(c)\} = \mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(c) = \beta_\alpha(a,c).\end{aligned}$$

Therefore β_α is a fuzzy equivalence relation. □

Corollary 2.3. *$\beta_\alpha(x_\alpha^{-1}, y_\alpha^{-1}) = \beta_\alpha(x,y)$ for all $x, y \in M$, where $x_\alpha^{-1}, y_\alpha^{-1}$ are inverses of x and y in M_α.*

Proof. μ_{H_α} is a fuzzy subgroup of M_α. Thus

$$\beta_\alpha(x_\alpha^{-1}, y_\alpha^{-1}) = \mu_{H_\alpha}(x_\alpha^{-1}) \wedge \mu_{H_\alpha}(y_\alpha^{-1}) = \mu_{H_\alpha}(x) \wedge \mu_{H_\alpha}(y) = \beta_\alpha,$$

which completes the proof. □

Proposition 2.4. *β_α is a fuzzy congruence on M.*

Proof. Indeed,

$$\begin{aligned}\beta_\alpha(a\alpha c, b\alpha d) &= \mu_{H_\alpha}(a\alpha c) \wedge \mu_{H_\alpha}(b\alpha d) \\ &\geqslant (\mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(c)) \wedge (\mu_{H_\alpha}(b) \wedge \mu_{H_\alpha}(d)) \\ &= (\mu_{H_\alpha}(a) \wedge \mu_{H_\alpha}(b)) \wedge (\mu_{H_\alpha}(c) \wedge \mu_{H_\alpha}(d)) \\ &= \beta_\alpha(a,b) \wedge \beta_\alpha(c,d).\end{aligned}$$

This completes the proof. □

Definition 2.5. If a fuzzy set is a (normal) fuzzy subgroup of M_α/H_α, then it is called a (*normal*) *fuzzy quotient* Γ-*subgroup*. For any normal subgroup H_α of M_α we define a fuzzy set $R : M_\alpha/H_\alpha \to [0,1]$ by putting $R(x\alpha H_\alpha) = \beta_\alpha(x,h)$ for all $h \in H_\alpha$.

Proposition 2.6. *R is a normal fuzzy quotient subgroup of M_α/H_α.*

Proof. Since μ_{H_α} is a fuzzy subgroup of M_α, for all $x\alpha H, y\alpha H \in M_\alpha/H_\alpha$ we have

$$\begin{aligned} R(x\alpha H_\alpha \alpha y\alpha H_\alpha) &= \beta_\alpha(x\alpha y, h) = \mu_{H_\alpha}(x\alpha y) \wedge \mu_{H_\alpha}(h) \\ &\geqslant (\mu_{H_\alpha}(x) \wedge \mu_{H_\alpha}(y)) \wedge \mu_{H_\alpha}(h) \\ &= \big(\mu_{H_\alpha}(x) \wedge \mu_{H_\alpha}(h)\big) \wedge \big(\mu_{H_\alpha}(y) \wedge \mu_{H_\alpha}(h)\big) \\ &= \beta_\alpha(x,h) \wedge \beta_\alpha(y,h) = R(x\alpha H) \wedge R(y\alpha H) \end{aligned}$$

and

$$\begin{aligned} R(x_\alpha^{-1}\alpha H_\alpha) &= \beta_\alpha(x_\alpha^{-1}, h) = \mu_{H_\alpha}(x_\alpha^{-1}) \wedge \mu_{H_\alpha}(h) \\ &\geqslant \mu_{H_\alpha}(x) \wedge \mu_{H_\alpha}(h) = \beta_\alpha(x,h) = R(x\alpha H_\alpha). \end{aligned}$$

Thus R ia a quotient fuzzy subgroup of M_α/H_α. Since μ_{H_α} is normal

$$\begin{aligned} R(x\alpha H_\alpha \alpha y\alpha H_\alpha) &= \beta_\alpha(x\alpha y, h) = \mu_{H_\alpha}(x\alpha y) \wedge \mu_{H_\alpha}(h) \\ &= \mu_{H_\alpha}(y\alpha x) \wedge \mu_{H_\alpha}(h) = \beta_\alpha(y\alpha x, h) = R(y\alpha H_\alpha \alpha x\alpha H_\alpha). \end{aligned}$$

Hence R is a normal quotient fuzzy subgroup of M_α/H_α. □

Proposition 2.7. *If M_α/H_α is finite and R is its fuzzy quotient subgroup, then R is a fuzzy subgroup.*

Proof. Since M_α/H_α is finite, every $x\alpha H_\alpha \in M_\alpha/H_\alpha$ has finite order, say n. Then $(x\alpha H_\alpha)^n = (x\alpha)^{n-1}x\alpha H_\alpha = H_\alpha$, where H_α is the identity of M_α/H_α. Thus $(x\alpha H_\alpha)^{-1} = x_\alpha^{-1}\alpha H_\alpha = (x\alpha)^{n-2}x\alpha H_\alpha$ and

$$\begin{aligned} R(x_\alpha^{-1}\alpha H_\alpha) &= R((x\alpha)^{n-2}x\alpha H_\alpha) = \beta_\alpha((x\alpha)^{n-2}x, h) \\ &= \mu_{H_\alpha}((x\alpha)^{n-3}x\alpha x) \wedge \mu_{H_\alpha}(h) = \mu_{H_\alpha}((x\alpha)^{n-3}x\alpha x) \\ &\geqslant \mu_{H_\alpha}((x\alpha)^{n-3}x) \wedge \mu_{H_\alpha}(x) \geqslant \mu_{H_\alpha}(x) \\ &= \mu_{H_\alpha}(x) \wedge \mu_{H_\alpha}(h) = \beta_\alpha(x,h) = R(x\alpha H_\alpha). \end{aligned}$$

Hence R is a fuzzy quotient subgroup. □

Proposition 2.8. *Let R be a fuzzy quotient subgroup of a group M_α/H_α and let $x\alpha H_\alpha \in M_\alpha/H_\alpha$. Then*

$$R(x\alpha H_\alpha \alpha y\alpha H_\alpha) = R(y\alpha H_\alpha) \Longleftrightarrow R(x\alpha H_\alpha) = R(H_\alpha).$$

Proof. If $R(x\alpha H_\alpha \alpha y\alpha H_\alpha) = R(y\alpha H_\alpha)$ holds for all $y\alpha H_\alpha \in M_\alpha/H_\alpha$, then putting $y\alpha H_\alpha = H_\alpha$, we obtain $R(x\alpha H_\alpha) = R(H_\alpha)$.

Conversely, suppose that $R(x\alpha H_\alpha) = R(H_\alpha)$. Since R is a fuzzy subgroup of M_α/H_α and μ_{H_α} is a fuzzy subgroup of M_α, we have

$$\begin{aligned} R(x\alpha H_\alpha \alpha y\alpha H_\alpha) &\geqslant R(x\alpha H_\alpha) \wedge R(y\alpha H_\alpha) = R(H_\alpha) \wedge R(y\alpha H_\alpha) \\ &= \beta_\alpha(e,h) \wedge \beta(y\alpha H_\alpha) = \mu_{H_\alpha}(h) \wedge \mu_{H_\alpha}(y) \\ &= \beta_\alpha(y,h) = R(y\alpha H_\alpha). \end{aligned}$$

Interchanging $x\alpha H_\alpha \alpha y\alpha H_\alpha$ with $y\alpha H_\alpha$, we get

$$R(y\alpha H_\alpha) \geqslant R(x\alpha H_\alpha \alpha y\alpha H_\alpha).$$

Hence the proof is completed. □

Proposition 2.9. *The intersection of two normal fuzzy quotient subgroups of M_α/H_α also is a normal fuzzy quotient subgroups of M_α/H_α.*

Proof. Let R and Q be two normal fuzzy quotient subgroups of M_α/H_α. Then for ally $x\alpha H_\alpha, y\alpha H_\alpha \in M_\alpha/H_\alpha$ we have

$$\begin{aligned} (R\cap Q)(x\alpha H_\alpha \alpha y\alpha H_\alpha) &= R(x\alpha H_\alpha \alpha y\alpha H_\alpha) \wedge Q(x\alpha H_\alpha \alpha y\alpha H_\alpha) \\ &\geqslant (R(x\alpha H_\alpha) \wedge R(y\alpha H_\alpha)) \wedge (Q(x\alpha H_\alpha) \wedge Q(y\alpha H_\alpha)) \\ &= (R(x\alpha H_\alpha) \wedge Q(x\alpha H_\alpha)) \wedge (R(y\alpha H_\alpha) \wedge Q(y\alpha H_\alpha)) \\ &= (R\cap Q)(x\alpha H_\alpha) \wedge (R\cap Q)(y\alpha H_\alpha) \end{aligned}$$

and

$$\begin{aligned} (R\cap Q)(x_\alpha^{-1}\alpha H_\alpha) &= R(x_\alpha^{-1}\alpha H_\alpha) \wedge Q(x_\alpha^{-1}\alpha H_\alpha) = R(x\alpha H_\alpha) \wedge Q(x\alpha H_\alpha) \\ &\leqslant (R\cap Q)(x\alpha H_\alpha). \end{aligned}$$

Interchanging $x\alpha H_\alpha$ with $x_\alpha^{-1}\alpha H_\alpha$, we obtain $(R\cap Q)(x\alpha H_\alpha) \leqslant (R\cap Q)(x_\alpha^{-1}\alpha H_\alpha)$. Hence $R\cap Q$ is a fuzzy subgroup of M_α/H_α. It is normal because

$$\begin{aligned} (R\cap Q)(x\alpha H_\alpha \alpha y\alpha H_\alpha) &= R(x\alpha H_\alpha \alpha y\alpha H_\alpha) \wedge Q(x\alpha H_\alpha \alpha y\alpha H_\alpha) \\ &= R(y\alpha H_\alpha \alpha x\alpha H_\alpha) \wedge Q(y\alpha H_\alpha \alpha x\alpha H_\alpha) \\ &\leqslant (R\cap Q)(y\alpha H_\alpha \alpha x\alpha H_\alpha). \end{aligned}$$

This completes the proof. □

Definition 2.10. On M_α/H_α we define a fuzzy relation $\mu_{\alpha,R}$ putting

$$\mu_{\alpha,R}(x\alpha H_\alpha, y\alpha H_\alpha) = R(x\alpha H_\alpha \alpha y_\alpha^{-1}\alpha H_\alpha)$$

for all $x\alpha H_\alpha, y\alpha H_\alpha \in M_\alpha/H_\alpha$.

Proposition 2.11. *$\mu_{\alpha,R}$ is a fuzzy congruence on M_α/H_α.*

Proof. It is clear that this relation is transitive. Since

$$\begin{aligned}\mu_{\alpha,R}(x\alpha H_\alpha, y\alpha H_\alpha) &= R(x\alpha H_\alpha \alpha y\alpha H_\alpha) = R((y\alpha x_\alpha^{-1})_\alpha^{-1}\alpha H_\alpha)\\ &= R(y\alpha x_\alpha^{-1}\alpha H_\alpha) = R(y\alpha H_\alpha \alpha x_\alpha^{-1}\alpha H_\alpha)\\ &= \mu_{\alpha,R}(y\alpha H_\alpha, x\alpha H_\alpha)\end{aligned}$$

it is also symmetric. Moreover, for all $x\alpha H_\alpha, y\alpha H_\alpha \in M_\alpha/H_\alpha$ we have

$$\begin{aligned}&(\mu_{\alpha,R} \circ \mu_{\alpha,R})(x\alpha H_\alpha, y\alpha H_\alpha)\\ &\quad= \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \{\mu_{\alpha,R}(x\alpha H_\alpha, z\alpha H_\alpha) \wedge \mu_{\alpha,R}(z\alpha H_\alpha, y\alpha H_\alpha)\}\\ &\quad= \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{R(x\alpha H_\alpha \alpha z_\alpha^{-1}\alpha H_\alpha) \wedge R(z\alpha H_\alpha \alpha y_\alpha^{-1}\alpha H_\alpha)\big\}\\ &\quad= \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{R(x\alpha z_\alpha^{-1}\alpha H_\alpha) \wedge R(z\alpha y_\alpha^{-1}\alpha H_\alpha)\big\}\\ &\quad= \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{\beta_\alpha(x\alpha z_\alpha^{-1}, h) \wedge \beta_\alpha(z\alpha y_\alpha^{-1}, h)\big\}\\ &\quad= \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{\big(\mu_{H_\alpha}(x\alpha z_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big) \wedge \big(\mu_{H_\alpha}(z\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big)\big\}\\ &\quad\leqslant \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{\big(\mu_{H_\alpha}(x\alpha z_\alpha^{-1}) \wedge \mu_{H_\alpha}(z\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big\}\\ &\quad\leqslant \bigvee_{z\alpha H_\alpha \in M_\alpha/H_\alpha} \big\{\mu_{H_\alpha}(x\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big\} = \mu_{H_\alpha}(x\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\\ &\quad= \beta_\alpha(x\alpha y_\alpha^{-1}, h) = R(x\alpha H_\alpha \alpha y_\alpha^{-1}\alpha H_\alpha) = \mu_{\alpha,R}(x\alpha H_\alpha, y\alpha H_\alpha).\end{aligned}$$

So, $\mu_{\alpha,R}$ is an equivalence relation.

To prove that it is a congruence observe that

$$\begin{aligned}&\mu_{\alpha,R}(x\alpha H_\alpha, y\alpha H_\alpha) \wedge \mu_{\alpha,R}(z\alpha H_\alpha, w\alpha H_\alpha)\\ &\quad= R(x\alpha H_\alpha \alpha y_\alpha^{-1}\alpha H_\alpha) \wedge R(z\alpha H_\alpha \alpha w_\alpha^{-1}\alpha H_\alpha)\\ &\quad= R(x\alpha y_\alpha^{-1}\alpha H_\alpha) \wedge R(z\alpha w_\alpha^{-1}\alpha H_\alpha)\\ &\quad= \beta_\alpha(x\alpha y_\alpha^{-1}, h) \wedge \beta_\alpha(z\alpha w_\alpha^{-1}, h)\\ &\quad= \{\big(\mu_{H_\alpha}(x\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big) \wedge \big(\mu_{H_\alpha}(z\alpha w_\alpha^{-1}) \wedge \mu_{H_\alpha}(h)\big)\}\\ &\quad= \mu_{H_\alpha}(x\alpha y_\alpha^{-1}) \wedge \mu_{H_\alpha}(z\alpha w_\alpha^{-1})\\ &\quad= \mu_{H_\alpha}(y_\alpha^{-1}\alpha x) \wedge \mu_{H_\alpha}(z\alpha w_\alpha^{-1}).\end{aligned}$$

Since μ_{H_α} is a fuzzy normal subgroup of M_α

$$\begin{aligned}\mu_{H_\alpha}(y_\alpha^{-1}\alpha x)\wedge\mu_{H_\alpha}(z\alpha w_\alpha^{-1}) \leqslant \mu_{H_\alpha}(y_\alpha^{-1}x\alpha z\alpha w_\alpha^{-1}) &= \mu_{H_\alpha}(x\alpha z\alpha w_\alpha^{-1}\alpha y_\alpha^{-1})\\ &= \mu_{\alpha,H_\alpha}(x\alpha z\alpha(y\alpha w)_\alpha^{-1})\wedge\mu_{\alpha,H_\alpha}(h)\\ &= \beta_\alpha(x\alpha z\alpha(y\alpha w)_\alpha^{-1},h)\\ &= R(x\alpha z\alpha H_\alpha\alpha(y\alpha w)_\alpha^{-1}\alpha H_\alpha)\\ &= \mu_{\alpha,R}(x\alpha z\alpha H_\alpha, y\alpha w\alpha H_\alpha),\end{aligned}$$

which completes the proof. □

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