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n-groups as n-groupoids with laws

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Abstract

In this article n-group (Q,A) is described as an n-groupoid (Q,B) in which the following two laws hold: $B(B(x,z,b_1^{n-2}),B(y,a_1^{n-2},z),a_1^{n-2}) = B(x,y,b_1^{n-2})$ and $B(a,c_1^{n-2},B(B(B(z,c_1^{n-2},z),c_1^{n-2},b),c_1^{n-2},B(B(z,c_1^{n-2},z),c_1^{n-2},a))) = b$.

1. Preliminaries

1.1. Definition. Let $n \geq 2$ and let (Q,A) be an n–groupoid. We say that (Q,A) is a *Dörnte n–group* (briefly: *n–group*) iff it is an n–semigroup and an n–quasigroup as well.

1.2. Proposition. ([17]) *Let $n \geq 2$ and let (Q,A) be an n–groupoid. Then the following statements are equivalent:*

(i) *(Q,A) is an n-group,*

(ii) *there are mappings ${}^{-1}$ and $\mathbf{e}$ respectively of the sets Q^{n-1} and Q^{n-2} into the set Q such that the following laws hold in the algebra $(Q,\{A,{}^{-1},\mathbf{e}\})$ (of the type $\langle n,n-1,n-2\rangle$)*

(a) $A(x_1^{n-2},A(x_{n-1}^{2n-2}),x_{2n-1}) = A(x_1^{n-1},A(x_n^{2n-1}))\,,$

(b) $A(\mathbf{e}(a_1^{n-2}),a_1^{n-2},x) = x\,,$

(c) $A((a_1^{n-2},a)^{-1},a_1^{n-2},a) = \mathbf{e}(a_1^{n-2})\,,$

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(*iii*) *there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *(of the type* $\langle n, n-1, n-2\rangle$*)*

($\bar{a}$) $A(A(x_1^n), x_{n+1}^{2n-1}) = A(x_1, A(x_2^{n+1}), x_{n+2}^{2n-1})$,

($\bar{b}$) $A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = x$,

($\bar{c}$) $A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = \mathbf{e}(a_1^{n-2})$.

1.3. Remarks. $\mathbf{e}$ is an $\{1, n\}$–neutral operation of n–groupoid (Q, A) iff algebra $(Q, \{A, \mathbf{e}\})$ of type $\langle n, n-2\rangle$ satisfies the laws (b) and ($\bar{b}$) from 1.2 (cf. [14]). The notion of $\{i, j\}$–neutral operation ($i, j \in \{1, ..., n\}$, $i < j$) of an n–groupoid is defined in a similar way (cf. [14]). Every n–groupoid has at most one $\{i, j\}$–neutral operation. In every n–group ($n \geq 2$) there is an $\{1, n\}$–neutral operation (cf. [14]). There are n–groups without $\{i, j\}$–neutral operation with $\{i, j\} \neq \{1, n\}$. In [16], n–groups with $\{i, j\}$–neutral operations, for $\{i, j\} \neq \{1, n\}$ are described. Operation $^{-1}$ from 1.2 is a generalization of the inverse operation in a group. In fact, if (Q, A) is an n–group, $n \geq 2$, then for every $a \in Q$ and for every sequence a_1^{n-2} over Q is

$$(a_1^{n-2}, a)^{-1} = \mathsf{E}(a_1^{n-2}, a, a_1^{n-2}),$$

where E is an $\{1, 2n-1\}$–neutral operation of the $(2n-1)$-group $(Q, \overset{2}{A})$, $\overset{2}{A}(x_1^{2n-1}) = A(A(x_1^n), x_{n+1}^{2n-1})$ (cf. [15]). (For $n = 2$, $a^{-1} = \mathsf{E}(a)$, a^{-1} is the inverse element of the element a with respect to the neutral element $\mathbf{e}(\emptyset)$ of the group (Q, A).)

1.4. Proposition. ([18]) *Let* $n \geq 2$ *and let* (Q, A) *be an* n*-groupoid. Then,* (Q, A) *is an* n*-group iff the following statements hold:*

(1) $(\forall x_i \in Q)_1^{2n-1} A(A(x_1^n), x_{n+1}^{2n-1}) = A(x_1^{n-1}, A(x_n^{2n-1}))$,

(2) $(\forall x_i \in Q)_1^{2n-1} A(x_1^{n-2}, A(x_{n-1}^{2n-2}), x_{2n-1}) = A(x_1^{n-1}, A(x_n^{2n-1}))$ *or*
$(\forall x_i \in Q)_1^{2n-1} A(A(x_1^n), x_{n+1}^{2n-1}) = A(x_1, A(x_2^{n+1}), x_{n+2}^{2n-1})$,

(3) *for every* $a_1^n \in Q$ *there is at least one* $x \in Q$ *and at least one* $y \in Q$ *such that* $A(a_1^{n-1}, x) = a_n$ *and* $A(y, a_1^{n-1}) = a_n$.

Note that the following proposition has been proved in [13]:
An n–semigroup (Q,A) is an n–group iff for each $a_1^n \in Q$ there exists at least one $x \in Q$ and at least one $y \in Q$ such that the following equalities hold: $A(a_1^{n-1}, x) = a_n$ and $A(y, a_1^{n-1}) = a_n$.

This assertion has been already formulated in [11], but the proof is missing there. W.A. Dudek has pointed my attention to this fact. Similar issues have been considered in [5] (Proposition 1).

1.5. Proposition. *Let $n \geq 3$ and let (Q,A) be an n-groupoid. Also let:*

(i) the $\langle 1,2\rangle$–associative law holds in (Q,A),

(ii) for every $x, y, a_1^{n-1} \in Q$ the following implication holds

$$A(x, a_1^{n-1}) = A(y, a_1^{n-1}) \Rightarrow x = y.$$

Then (Q,A) is an n-semigroup.

Proposition 1.5 is a part of proposition 3.5 from [17]. In the proof of this proposition we use the method of E. I. Sokolov from [11].

1.6. Proposition. *Let (Q,A) be an n–group, $^{-1}$ its inverse operation, $\mathbf{e}$ its $\{1,n\}$–neutral operation and $n \geq 2$. Also let*

$$^{-1}\!A(x, a_1^{n-2}, y) = z \overset{def}{\Longleftrightarrow} A(z, a_1^{n-2}, y) = x$$

for all $x, y, z \in Q$ and for every sequence a_1^{n-2} over Q. Then, for all $x, y \in Q$ and for every sequence a_1^{n-2} over Q the following equalities hold:

$(\bar{1})$ $^{-1}\!A(x, a_1^{n-2}, y) = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1})$,

$(\bar{2})$ $\mathbf{e}(a_1^{n-2}) = {}^{-1}\!A(x, a_1^{n-2}, x)$,

$(\bar{3})$ $(a_1^{n-2}, x)^{-1} = {}^{-1}\!A({}^{-1}\!A(x, a_1^{n-2}, x), a_1^{n-2}, x)$,

$(\bar{4})$ $A(x, a_1^{n-2}, y) = {}^{-1}\!A(x, a_1^{n-2}, {}^{-1}\!A({}^{-1}\!A(y, a_1^{n-2}, y), a_1^{n-2}, y))$.

Sketch of the proof.

a) $^{-1}A(x, a_1^{n-2}, y) = z \Longleftrightarrow A(z, a_1^{n-2}, y) = x \Longleftrightarrow$
$A(A(z, a_1^{n-2}, y), a_1^{n-2}, (a_1^{n-2}, y)^{-1}) = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1}) \Longleftrightarrow$
$A(z, a_1^{n-2}, A(y, a_1^{n-2}, (a_1^{n-2}, y)^{-1})) = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1}) \Longleftrightarrow$
$A(z, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1}) \Longleftrightarrow$
$z = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1})$.

b) $^{-1}A(x, a_1^{n-2}, x) = \mathbf{e}(a_1^{n-2}) \Longleftrightarrow A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = x$.

c) $^{-1}A(^{-1}A(x, a_1^{n-2}, x), a_1^{n-2}, x) = (a_1^{n-2}, x)^{-1} \Longleftrightarrow$
$A((a_1^{n-2}, x)^{-1}, a_1^{n-2}, x) = {}^{-1}A(x, a_1^{n-2}, x) \Longleftrightarrow$
$A((a_1^{n-2}, x)^{-1}, a_1^{n-2}, x) = \mathbf{e}(a_1^{n-2})$.

d) $A(x, a_1^{n-2}, y) = {}^{-1}A(x, a_1^{n-2}, {}^{-1}A(^{-1}A(y, a_1^{n-2}, y), a_1^{n-2}, y)) \Longleftrightarrow$
$x = A(A(x, a_1^{n-2}, y), a_1^{n-2}, (a_1^{n-2}, y)^{-1}) \Longleftrightarrow$
$x = A(x, a_1^{n-2}, A(y, a_1^{n-2}, (a_1^{n-2}, y)^{-1})) \Longleftrightarrow$
$x = A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2}))$. □

2. Results

2.1. Theorem. *Let $n \geq 2$ and let (Q, A) be an n–group. Furthermore, let $B = {}^{-1}A$, where*

$$^{-1}A(x, z_1^{n-2}, y) = z \Longleftrightarrow A(z, z_1^{n-2}, y) = x$$

for all $x, y, z \in Q$ and for every sequence z_1^{n-2} over Q. Then the following laws

(i) $B(B(x, z, b_1^{n-2}),\ B(y, a_1^{n-2}, z), a_1^{n-2}) = B(x, y, b_1^{n-2})$,

(ii)

$$B(a, c_1^{n-2}, B(B(B(z, c_1^{n-2}, z), c_1^{n-2}, b), c_1^{n-2}, B(B(z, c_1^{n-2}, z), c_1^{n-2}, a))) = b$$

hold in the n–groupoid (Q, B). Moreover, for all $x, y \in Q$ and for every sequence a_1^{n-2} over Q the following equality holds

$$B(x, a_1^{n-2}, y) = A(x, a_1^{n-2}, (a_1^{n-2}, y)^{-1}),$$

where $^{-1}$ *is an inverse operation of the* n*–group* (Q,A).

Proof. Let $n \geq 2$ and let (Q,A) be an n–group, $^{-1}$ its inverse operation and $\mathbf{e}$ its $\{1,n\}$–neutral operation. Also let

$$(0) \qquad {}^{-1}A(x,z_1^{n-2},y)=z \overset{def}{\Longleftrightarrow} A(z,z_1^{n-2},y)=x$$

for all $x,y,z\in Q$ and for every sequence z_1^{n-2} over Q.

1) By 1.1 and (0), we conclude that for all $x,y,z,u,v\in Q$, for every sequence a_1^{n-2} over Q and for every sequence b_1^{n-2} over Q the following series of implications holds

$$A(A(x,y,a_1^{n-2}),z,b_1^{n-2})=A(x,A(y,a_1^{n-2},z),b_1^{n-2}) \Longrightarrow$$
$${}^{-1}A(A(x,A(y,a_1^{n-2},z),b_1^{n-2}),z,b_1^{n-2})=A(x,y,a_1^{n-2}) \Longrightarrow$$
$${}^{-1}A(A(x,u,b_1^{n-2}),z,b_1^{n-2})=A(x,\,{}^{-1}A(u,a_1^{n-2},z),a_1^{n-2}) \Longrightarrow$$
$${}^{-1}A(v,z,b_1^{n-2})=A(\,{}^{-1}A(v,u,b_1^{n-2}),\,{}^{-1}A(u,a_1^{n-2},z),a_1^{n-2}) \Longrightarrow$$
$${}^{-1}A(v,u,b_1^{n-2})=\,{}^{-1}A(\,{}^{-1}A(v,z,b_1^{n-2}),\,{}^{-1}A(u,a_1^{n-2},z),a_1^{n-2}).$$

But

$$A(y,a_1^{n-2},z)=u \Longleftrightarrow y=\,{}^{-1}A(u,a_1^{n-2},z),\, A(x,u,b_1^{n-2})=v \Longleftrightarrow$$
$$\Longleftrightarrow x=\,{}^{-1}A(v,u,b_1^{n-2}).$$

Whence, by the substitution $B=\,{}^{-1}A$, we conclude that

$$B(B(x,z,b_1^{n-2}),B(y,a_1^{n-2},z),a_1^{n-2})=B(x,y,b_1^{n-2})$$

holds in the n–groupoid (Q,B).

2) By 1.1, 1.2, 1.3 and (0), we conclude that for all $a,b,x\in Q$ and for every sequence c_1^{n-2} over Q the following series of equivalences holds

$${}^{-1}A(a,c_1^{n-2},x)=b \Longleftrightarrow A(b,c_1^{n-2},x)=a \Longleftrightarrow$$
$$A((c_1^{n-2},b)^{-1},c_1^{n-2},A(b,c_1^{n-2},x))=A((c_1^{n-2},b)^{-1},c_1^{n-2},a) \Longleftrightarrow$$
$$x=A((c_1^{n-2},b)^{-1},c_1^{n-2},a) \Longleftrightarrow$$
$$A(x,c_1^{n-2},(c_1^{n-2},a)^{-1})=A(A((c_1^{n-2},b)^{-1},c_1^{n-2},a),c_1^{n-2},(c_1^{n-2},a)^{-1}) \Longleftrightarrow$$
$$A(x,c_1^{n-2},(c_1^{n-2},a)^{-1})=(c_1^{n-2},b)^{-1} \Longleftrightarrow$$
$${}^{-1}A((c_1^{n-2},b)^{-1},c_1^{n-2},(c_1^{n-2},a)^{-1})=x \Longleftrightarrow$$
$${}^{-1}A(\,{}^{-1}A(\,{}^{-1}A(z,c_1^{n-2},z),c_1^{n-2},b),c_1^{n-2},\,{}^{-1}A(\,{}^{-1}A(z,c_1^{n-2},z),c_1^{n-2},a))=x.$$

But
$(c_1^{n-2},c)^{-1} = {}^{-1}\!A(\mathbf{e}(c_1^{n-2}),c_1^{n-2},c) \Longleftrightarrow \mathbf{e}(c_1^{n-2}) = A((c_1^{n-2},c)^{-1},c_1^{n-2},c)$
and

$$\mathbf{e}(c_1^{n-2}) = {}^{-1}\!A(z,c_1^{n-2},z) \Longleftrightarrow z = A(\mathbf{e}(c_1^{n-2}),c_1^{n-2},z)\,.$$

Whence, by the substitution $B = {}^{-1}\!A$, we conclude that (ii) holds in the n–groupoid (Q,B).

3) By the substitution $B = {}^{-1}\!A$ and by Proposition 1.6, we conclude that for all $x,y\in Q$ and for every sequence a_1^{n-2} over Q the following equality holds

$$B(x,a_1^{n-2},y) = A(x,a_1^{n-2},(a_1^{n-2},y)^{-1}\,)\,. \qquad \square$$

2.2. Theorem. *Let $n\geq 2$ and let (Q,B) be an n–groupoid in which the laws (i) and (ii) from the previous theorem holds. Then, there is an n-group (Q,A) such that ${}^{-1}\!A = B$. Moreover, for all $x,y\in Q$ and for every sequence a_1^{n-2} over Q the following equalities hold*

$$\mathbf{e}(a_1^{n-2}) = B(x,a_1^{n-2},x)\,,$$
$$(a_1^{n-2},x)^{-1} = B(B(x,a_1^{n-2},x),a_1^{n-2},x)\,,$$
$$A(x,a_1^{n-2},y) = B(x,a_1^{n-2},B(B(y,a_1^{n-2},y),a_1^{n-2},y))\,,$$

where ${}^{-1}$ is an inverse operation, and $\mathbf{e}$ *is an $\{1,n\}$–neutral operation of the n–group (Q,A).*

Proof. By (ii), we conclude that the following statement holds:

1^o For every $a_1^n\in Q$ there is at least one $x\in Q$ such that

$$B(a_1^{n-1},x) = a_n\,.$$

Furthermore, the following statements hold:

2^o $(\forall a\in Q)\,(\forall z\in Q)\,(\forall c_i\in Q)_1^{n-2}\quad B(a,B(z,c_1^{n-2},z),c_1^{n-2}) = a\,.$

3^o For every $a_1^n\in Q$ there is exactly one $y\in Q$ such that

$$B(y,a_1^{n-1}) = a_n\,.$$

4^o There exists n–ary operation ${}^{-1}\!B$ in Q such that for all $x,y\in Q$ and for every sequence a_1^{n-1} over Q

$(\bar{o})$ $$^{-1}B(x, a_1^{n-1}) = y \Longleftrightarrow B(y, a_1^{n-1}) = x\,.$$

5^o For every $a_1^n \in Q$ there is exactly one $y \in Q$ such that
$$^{-1}B(y, a_1^{n-1}) = a_n\,.$$

6^o For every $a_1^n \in Q$ there is at least one $x \in Q$ such that
$$^{-1}B(a_1^{n-1}, x) = a_n\,.$$

7^o The $\langle 1,2\rangle$–associative law holds in $(Q,\ ^{-1}B)$.

8^o $(Q,\ ^{-1}B)$ is an n–semigroup.

Sketch of the proof of 2^o.

a) $n \geq 3$. Putting $z = y$ in (i) we obtain
$$B(B(x, y, b_1^{n-2}), B(y, a_1^{n-2}, y), a_1^{n-2}) = B(x, y, b_1^{n-2})$$
which together with 1^o gives
$$(\forall x, y \in Q)\,(\forall b_i \in Q)_1^{n-3}\ (\forall a \in Q)\ (\exists b_{n-2} \in Q)\ \ B(x, y, b_1^{n-2}) = a\,.$$

b) $n = 2$. As in the previous case from (i) we obtain
$$B(B(x, y), B(y, y)) = B(x, y)\,,$$
which for $B(x, y) = a$ (by 1^o) proves that
$$(\forall x \in Q)\,(\forall a \in Q)\,(\exists y \in Q)\ \ B(a, B(y, y)) = a\,,$$
$$(\forall y \in Q)\,(\forall u \in Q)\,(\exists c \in Q)\ \ y = B(u, c)\,,$$
$$B(y, y) = B(B(u, c), B(u, c)) = B(u, u)\,,$$
which completes the proof of 2^o.

Sketch of the proof of 3^o and 4^o.

a) $B(x, a, b_1^{n-2}) = B(y, a, b_1^{n-2}) \implies$
$$B(B(x, a, b_1^{n-2}), B(u, a_1^{n-2}, a), a_1^{n-2}) = B(B(y, a, b_1^{n-2}), B(u, a_1^{n-2}, a), a_1^{n-2})$$
$$\implies B(x, u, b_1^{n-2}) = B(y, u, b_1^{n-2})\,.$$

Now, putting $u = A(v, b_1^{n-2}, v)$ and using 2^o, we obtain
$$B(x, B(v, b_1^{n-2}, v), b_1^{n-2}) = B(y, B(v, b_1^{n-2}, v), b_1^{n-2}) \implies x = y.$$

b) $B(x,a,b_1^{n-2}) = c \Longleftrightarrow$

$$B(B(x,a,b_1^{n-2}), B(u,a_1^{n-2},a), a_1^{n-2}) = B(c, B(u,a_1^{n-2},a), a_1^{n-2}) \Longleftrightarrow$$

$$B(x,u,b_1^{n-2}) = B(c, B(u,a_1^{n-2},a), a_1^{n-2})$$

by (i). Putting $u = A(v,b_1^{n-2},v)$ we obtain

$$B(x, B(v,b_1^{n-2},v), b_1^{n-2}) = B(c, B(B(v,b_1^{n-2},v), a_1^{n-2}, a), a_1^{n-2}),$$

which (by 2^o) is equivalent to

$$x = B(c, B(B(v,b_1^{n-2},v), a_1^{n-2}, a), a_1^{n-2}).$$

Sketch of the proof of 5^o.

$$^{-1}B(x,c_1^{n-1}) = u \Longleftrightarrow B(u,c_1^{n-1}) = x,$$

$$^{-1}B(y,c_1^{n-1}) = v \Longleftrightarrow B(v,c_1^{n-1}) = y.$$

Thus

$$x = y \Longrightarrow u = v \text{ and } u = v \Rightarrow x = y.$$

Sketch of the proof of 6^o.

$$^{-1}B(a,a_1^{n-2},x) = b \Longleftrightarrow B(b,a_1^{n-2},x) = a.$$

Sketch of the proof of 7^o.

$$B(v,u,b_1^{n-2}) = B(B(v,z,b_1^{n-2}), B(u,a_1^{n-2},z), a_1^{n-2}) \Longrightarrow$$

$$B(v,z,b_1^{n-2}) = {}^{-1}B(B(v,u,b_1^{n-2}), B(u,a_1^{n-2},z), a_1^{n-2}) \Longrightarrow$$

$$B({}^{-1}B(x,u,b_1^{n-2}), z, b_1^{n-2}) = {}^{-1}B(x, B(u,a_1^{n-2},z)a_1^{n-2}) \Longrightarrow$$

$$B({}^{-1}B(x, {}^{-1}B(y,a_1^{n-2},z), b_1^{n-2}), z, b_1^{n-2}) = {}^{-1}B(x,y,a_1^{n-2}) \Longrightarrow$$

$$^{-1}B({}^{-1}B(x,y,a_1^{n-2}), z, b_1^{n-2}) = {}^{-1}B(x, {}^{-1}B(y,a_1^{n-2},z), b_1^{n-2}).$$

Since

$$B(v,u,b_1^{n-2}) = x \Longleftrightarrow {}^{-1}B(x,u,b_1^{n-2}) = v$$

and

$$B(u,a_1^{n-2},z) = y \Longleftrightarrow {}^{-1}B(y,a_1^{n-2},z) = u.$$

Sketch of the proof of 8^o.

The case $n = 2$ follows from 7^o. The case $n \geq 3$ is a consequence of 7^o, 5^o and 1.5.

Now, by 5^o, 6^o, 8^o, 1.4, $(\bar{o})$ and the substitution $A = {}^{-1}B$, we conclude that (Q, A) is an n–group. Hence, 1.3 and 1.6 completes the proof. □

2.3. Remark. In this paper n–group (Q, A), $n \geq 2$, is described as an n–groupoid $(Q, {}^{-1}A)$ with two laws. Similarly, the n–group (Q, A) can be described as the n–groupoid (Q, A^{-1}) such that

$$A^{-1}(x, a_1^{n-2}, y) = z \Longleftrightarrow A(x, a_1^{n-2}, z) = y\,.$$

Variety of groups of the type $\langle 2 \rangle$ has been considered in [7] (see, also [8] and [3]). The investigation of this paper was extended in [12] for groups, for rings and, more generally, for Ω-groups. In [6] group is described as an groupoid (Q, B) which satisfies one law (i.e. our (i) for $n = 2$) and in which the equality $B(a, x) = b$ has at least one solution x for each $a, b \in Q$.

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