Linear isotopes of small order groups

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Abstract

The first part of the results of computer investigation of linear group isotopes is given. The first section of the work is auxiliary. It contains beloning criterions to the known classes of quasigroups. The second section contains: an algorithm for description of linear group isotopes; a full list of pairwise non-isomorphic linear group isotopes up to 15 order; a full list of subquasigroups of every isotope. For each quasigroup the beloning to the known classes of quasigroups is singled out.

A groupoid $(G;\cdot)$ is called an *isotope of a groupoid* $(Q;+)$, iff there exists a triple (α,β,γ) of bijections, called an *isotopy*, such that the relation

$$\gamma(x\cdot y)=\alpha x+\beta y$$

holds. An isotope of a group is called a *group isotope*. An isotope of a group is called *linear* if every component of a corresponding isotopy is a linear transformation of the group (recall, a transformation α is said to be *linear* in the group $(Q;+)$, iff there exist an automorphism of the group and an element c such that $\alpha x=\theta x+c$ for all $x\in Q$). It is easy to verify that any groupoid isomorphic to a linear group isotope is a linear group isotope as well. So, the class of all linear group isotopes forms a variety and medial and T-quasigroups are its subvarieties. An additional information on group isotopes and linear group isotopes one can find in [1], [2] and [3].

Here, using the results of the work [3] we continue that study. Namely, in the first part of the article we give: a criterion for a linear group isotope to belong to each of 22 the most singnificant classes of quasigroups; a full list of pairwise nonisomorphic linear group isotopes up to 15 order; a number of all these isotopes of every order (≤ 15).

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1. Some necessary properties

We shall write "isotope (α,β,γ) of the groupoid $(Q;+)$" instead of isotope $(Q;*)$, defined by the equality $x*y=\gamma^{-1}(\alpha x+\beta y)$", where α, β, γ are substitutions of the set Q.

Up to isomorphism every isotope $(Q;\cdot)$ of a group $(Q;+)$ can be defined by the equality

$$xy=\varphi x+\psi y+c, \tag{1}$$

where φ, ψ are unitary substitutions of the group $(Q;+)$ (i.e. $\varphi 0=\psi 0=0$). If $(Q;\cdot)$ is linear, then φ, ψ are automorphisms of $(Q;+)$ (see [3] and **Theorem 1.6** here).

As usual, the signs f_a and e_a denote a left and right local units of a of the quasigroup operation $(\cdot)$:

$$f_a\cdot a=a\cdot e_a=a;$$

ε denotes the identical transformation of arbitrary fixed set. I_c denotes inner automorphism of the group operation $(+)$:

$$I_c x = -c + x + c.$$

A *commutation of the arbitrary operation* $(+)$ is denoted by $(\oplus)$ and defined by $x \oplus y = y + x$. A groupoid $(Q;\oplus)$ is said *to be a commutation* of a groupoid $(Q;+)$.

A class of groupoids will be called *commutation of a class* K *of groupoids* and will be denoted by K^*, if K^* consists of all commutations of groupoids from K.

A formula $\Phi(\varphi,\psi,c,+,I,\bullet,J,A)$, being an equality with no propositional constant and having the propositional variables $\varphi,\psi,c,x_1,\dots,x_n$, binary functional variables $+,\bullet,A$ and unary ones I,J only, will be called *a beloning criterion of a linear isotope to a class* K, where K is some class of groupoids, if from the facts that c is an element of a group $(Q;+)$; φ,ψ are elements of the group $(H,\bullet) = Aut(Q,+)$; I,J are inverse operations in these two groups respectively; and $A: H \times Q \to Q$ is such a function, that $A(\alpha,x) = \alpha x$ is true, it follows the equivalence of predicate expressed by the formula $\forall x_1 \dots x_n\, \Phi(\varphi,\psi,c,+,I,\bullet,J,A)$ to the beloning of the linear isotope (φ,ψ,c) of the group $(Q;+)$ to the class K.

Theorem 1.1. *If* $\Phi(\varphi,\psi,c,+,I,\bullet,J,A)$ *is a beloning criterion of a linear isotope to a class* K *of groupoids, then a beloning criterion of a linear isotope to the class* K^* *one can get by replacing every subterm of the type* $u+v$, *where* u,v *are arbitrary terms, with* $v+u$ *in the formula* $\Phi(I_c\psi, I_c\varphi, c,+,I,\bullet,J,A)$.

Proof. Let c be an element of a group $(Q;+)$, φ,ψ be elements of the group $(H,\bullet)=Aut(Q,+)$, I,J be the inverse operations in these groups and

$$A: H\times Q\to Q$$

be a function such that $A(\alpha,x)=\alpha x$. Then the predicate expressed by the formula $\forall x_1...x_n\,\Phi(\varphi,\psi,c,+,I,\bullet,J,A)$ is equivalent to the beloning of linear isotope (φ,ψ,c) of the group $(Q;+)$ to a class K. Note, that c is an element of the group $(Q;\oplus)$ as well, the inverse in the groups $(Q;+)$ and $(Q;\oplus)$ for any element from the set Q is the same, and an automorphism groups of these groups coincide.

Hence, a predicate, expressed by a formula obtained in a way described in the theorem, coincide with the predicate, expressed by the formula $\forall x_1...x_n\,\Phi(I_c\psi,I_c\varphi,c,\oplus,I,\bullet,J,A)$, and is equivalent to the beloning of the linear isotope $(I_c\psi,I_c\varphi,c)$ of the group $(Q;\oplus)$ to the class K. But if $(Q;\cdot)$ is such an isotope, then

$$xy=I_c\psi x\oplus I_c\varphi y\oplus c=c+I_c\varphi y+I_c\psi x,$$

whence

$$x\otimes y=c+I_c\varphi x+I_c\psi y=\varphi x+\psi y+c,$$

i.e. this is equivalent to the beloning of the linear isotope (φ,ψ,c) of the group $(Q;+)$ to the class K^*. Thus, the **theorem** is true. □

Lemma 1.2.

$$(K_1\cap...\cap K_n)^*=K_1^*\cap...\cap K_n^*,\quad (K_1\cup...\cup K_n)^*=K_1^*\cup...\cup K_n^*.$$

Lemma 1.3. *If for automorphisms* φ,ψ *of a group* $(Q;+)$ *the equality*

$$\varphi + \psi = \varepsilon$$

holds, then the group is abelian.

Proof. Really, if for some elements $u, v \in Q$ $u + v \neq v + u$, then

$$\psi^{-1}u + \varphi^{-1}v = (\varphi + \psi)\psi^{-1}u + (\varphi + \psi)\varphi^{-1}v =$$
$$= \varphi\psi^{-1}u + u + v + \psi\varphi^{-1}v \neq \varphi\psi^{-1}u + v + u + \psi\varphi^{-1}v =$$
$$= (\varphi + \psi)(\psi^{-1}u + \varphi^{-1}v) = \psi^{-1}u + \varphi^{-1}v.$$

A contradiction. □

Note, that only in abelian group the mapping I ($Ix = -x$) is an automorphism.

Recall the following

Definition 1. By *right F-, right symmetrical, RIP-, right* ***Bol****, right distributive, right semimedial, right alternative quasigroup* and by *right loop* it is called a groupoid $(Q,\cdot)$, such that its commutation $(Q,\otimes)$ has the left respective condition. By *F-*, *TS-*, *IP-*, ***Bol****, distributive, semimedial, alternative quasigroup* is called a quasigroup fulfilling the left and right respective condition simultaneously. An idempotent *TS*-quasigroup (**Moufang**) is called *Steiner* (respectively *CH-*) quasigroup. A quasigroup having at least one idempotent element is said *to be a peak*. If a quasigroup has no subquasigroup exept itself then it is called *monoquasigroup*.

Theorem 1.4. *A belonging criterion of a linear isotope* (φ, ψ, c) *of a group* $(Q;+)$ *to a class of quasigroups* K *is defined by the following table*

N	name	definition	criterion
1	*commutative*	$xy = yx$	$\varphi = \psi$, *the group is abelian*
2	*medial*	$xy \cdot uv = xu \cdot yv$	$\varphi\psi = \psi\varphi$, *the group is abelian*
3	*idempotent*	$xx = x$	$\varphi + \psi = \varepsilon$, $c = 0$, *the group is abelian*
4	*left F*	$x \cdot yz = xy \cdot e_x z$	φ *commutes with* ψ *and with all inner automorphisms of the group*
5	*right F*	*see* ***Definition 1***	$I_c\psi$ *commutes with* φ *and with all inner automorphisms of the group*
6	*left symmetrical*	$x \cdot xy = y$	$\psi = -\varepsilon$, *the group is abelian*
7	*right symmetrical*	*see* ***Definition 1***	$\varphi = -\varepsilon$, *the group is abelian*
8	*LIP*	*exists a substitution* λ, *for which* $\lambda x \cdot xy = y$	$(I_c\psi)^2 = \varepsilon$
9	*RIP*	*see* ***Definition 1***	$\varphi^2 = \varepsilon$
10	***Mufang***	$(xy \cdot z)y = x \cdot y(e_y z \cdot y)$, $y(x \cdot yz) = (y \cdot xf_y)y \cdot z$	$\varphi^2 = (I_c\psi)^2 = \varepsilon$
11	*left* ***Bol***	$z(x \cdot zy) = R_{e_z}^{-1}(z \cdot xz) \cdot y$	$(I_c\psi)^2 = \varepsilon$
12	*right* ***Bol***	*see* ***Definition 1***	$\varphi^2 = \varepsilon$
13	*left distributive*	$x \cdot yz = xy \cdot xz$	$\varphi + \psi = \varepsilon$, $c = 0$, *the group is abelian*
14	*right distributive*	*see* ***Definition 1***	$\varphi + \psi = \varepsilon$, $c = 0$, *the group is abelian*
15	*left loop*	$f_x = f_y$	$I_c\psi = \varepsilon$
16	*right loop*	*see* ***Definition 1***	$\varphi = \varepsilon$
17	*left semimedial*	$xx \cdot yz = xy \cdot xz$	$\varphi\psi = \psi\varphi$, *the group is abelian*
18	*right semimedial*	*see* ***Definition 1***	$\varphi\psi = \psi\varphi$, *the group is abelian*
19	*primary*	*commutative, semimedial,* $yy \cdot yx = xx \cdot xx$	$\varphi = \psi$, $(\forall x)(3x = 0)$, *the group is abelian*
20	*left alternative*	$x \cdot xz = xx \cdot z$	$\varphi = I_c\psi = \varepsilon$
21	*right alternative*	*see* ***Definition 1***	$\varphi = I_c\psi = \varepsilon$
22	*elastic*	$xy \cdot x = x \cdot yx$	$\varphi\psi = \psi\varphi$, $\varphi c = \psi c$, $(\forall x, u)(I_u\varphi(-x + \varphi x) = \psi(\psi x - I_c^{-1}x))$

Proof. The points 1)-3) are evident. The points 6), 8), 11), 15), 18) follow from the points 7), 9), 12), 16), 17) correspondingly when applying **Theorem 1.1** and **Lemma 1.2**.

4) The identity $x\cdot yz = xy\cdot e_x z$ is equivalent to

$$\varphi x+\psi(\varphi y+\psi z+c)+c=\varphi(\varphi x+\psi y+c)+$$
$$+\psi(\varphi\psi^{-1}(-\varphi x+x-c)+\psi z+c)+c,$$

i.e.

$$\varphi x+\psi\varphi y=\varphi^2 x+\varphi\psi y+\varphi c-\psi\varphi\psi^{-1}\varphi x+\psi\varphi\psi^{-1}x-\psi\varphi\psi^{-1}c,$$

this with $x=0$ implies

$$\psi\varphi y=\varphi\psi y+\varphi c-\psi\varphi\psi^{-1}c,$$

whence $\varphi c=\psi\varphi\psi^{-1}c$ and $\psi\varphi=\varphi\psi$. Hence,

$$\varphi x+\varphi\psi y=\varphi^2 x+\varphi\psi y+\varphi c-\varphi^2 x+\varphi x-\varphi c,$$

i.e.

$$x+\psi y+c=\varphi x+\psi y+c-\varphi x+x,$$

or

$$x+v-x=\varphi x+v-\varphi x,$$

whence $I_x v=\varphi I_x(\varphi^{-1}v)$. Hence, $I_x\varphi=\varphi I_x$.

5) By **Theorem 1.1**, **Lemma 1.2** and the just proved point 4) the identity $xy\cdot z=xf_z\cdot yz$ is equivalent to a conjuction of the commutation of the automorphisms $I_c\varphi, I_c\psi$ and the equality

$$-x+v+x=-I_c\psi x+v+I_c\psi x.$$

From the second condition it follows that

$$-x+I_c\psi u+x=I_c\psi(-x+u+x),$$

i.e. $I_x I_c\psi=I_x\psi I_c$, this with $x=-c$ implies $\psi I_c=I_c\psi$. Applying the first condition we have $I_c\varphi I_c\psi=I_c^2\psi\varphi$, i.e. $\varphi I_c\psi=I_c\psi\varphi$.

9) The parameter identity $xy\cdot\rho y=x$ is equivalent to a conjuction of the equalities $\varphi^2=\varepsilon$ and $\rho=\psi^{-1}II_c R_{\varphi c}\varphi\psi$. Really, if the identity holds and $y=0$, then, accounting (1), we have $\varphi^2 x+a=x$

for some $a \in Q$. This means that $\varphi^2 = \varepsilon$ and $a = 0$. So, the identity can be rewritten as

$$x + \varphi\psi y + \varphi c + \psi\rho y + c = x.$$

Consequently, $\rho = \psi^{-1} I L_c R_{\varphi c} \varphi\psi$. Conversly, let the above equalities hold, then

$$xy \cdot \rho y = \varphi(\varphi x + \psi y + c) + \psi\psi^{-1}(-\varphi c - \varphi\psi y - c) + c =$$
$$= x + \varphi\psi y + \varphi c - \varphi c - \varphi\psi y - c + c = x.$$

7) To the proof of the point 9) we have to add that $\rho = \varepsilon$ i.e. $\varphi = -\varepsilon = I$ (since, as it is easy to see, $\varphi c = -c$). It means, in particular, that the group is abelian.

10) From the first equality with $y = z = 0$ and accounting (1) we have that

$$\varphi^3 x + a = \varphi x + b$$

for all $x \in Q$ and for some elements $a, b \in Q$. It is easy to see that the last equality is equivalent to $a = b$ and $\varphi^3 = \varphi$. So, $\varphi^2 = \varepsilon$ By **Theorem 1.1** from the second equality and from the just proved assertion we have $(I_c\psi)^2 = \varepsilon$. It is easy to verify that the converse statement is true as well.

12) The right **Bol** identity is the following:

$$(yz \cdot x)z = y \cdot L_{f_z}^{-1}(zx \cdot z).$$

When $x = z = 0$, we have a relation $\varphi^3 y + a = \varphi y + b$ for all $y \in Q$ and for some elements $a, b \in Q$. So, $\varphi^2 = \varepsilon$. It is easy to verify that the converse statement is true as well.

13), 14) The quasigroup is idempotent, so, $\varphi + \psi = \varepsilon$, $c = 0$ and the group is abelian. The converse statement is evident.

16) The relation (1) implies $e_x = \psi^{-1}(-\varphi x + x + c)$. The existence of a right identity means that $e_x = e_y$ for all $x, y \in Q$. It is easy to see, that the equality $e_x = e_0$ is equivalent to $\varphi = \varepsilon$.

17) In the linear group isotope the left semimedial identity is equivalent to

$$\varphi\psi x + \varphi c + \psi\varphi y = \varphi\psi y + \varphi c + \psi\varphi x, \qquad (2)$$

this with $y = 0$ implies $\psi\varphi = \varphi I_c \psi$. Then the equality (2) will be rewritten as $x + y = y + x$, i.e. $(Q;+)$ is abelian. Hence, $I_c = \varepsilon$, and then $\varphi\psi = \psi\varphi$.

19) By 1), 17), 18) $\varphi = \psi$ and $(Q;+)$ is abelian. Then the identity

$$yy \cdot yx = xx \cdot xx$$

is equivalent to $3\varphi^2 y = 3\varphi^2 x$, or $3y = 3x$. Replacing with $y = 0$, we have

$$(\forall x)(3x = 0).$$

That, in particular, means the truth of the equality $3y = 3x$.

20) The identity $x \cdot xz = xx \cdot z$ is equivalent to

$$\varphi x + \psi\varphi x + \psi u + c = \varphi^2 x + \varphi\psi x + \varphi c + u$$

If $x = u = 0$, then $\varphi c = c$ and then the additional equality $x = 0$ gives the relation $\psi u + c = c + u$, i.e. $I_c \psi = \varepsilon$. Since $\psi = I_c^{-1}$, then

$$\varphi x + I_c^{-1}\varphi x + I_c^{-1} u + c = \varphi^2 x + \varphi I_c^{-1} x + \varphi c + u,$$

i.e. $\varphi x + c = \varphi^2 x + \varphi c$. Accounting $\varphi c = c$ we have $\varphi = \varepsilon$, whence, in particular, it follows that $\varphi c = c$.

21) By 15), 16), 20) the left alternativity (and then right alternativity) of linear group isotopes may hold exactly for loops.

22) The identity $xy \cdot x = x \cdot yx$ is equivalent to

$$\varphi^2 x + \varphi\psi y + \varphi c + \psi x = \varphi x + \psi\varphi y + \psi^2 x + \psi c$$

If $x=y=0$, then $\varphi c=\psi c$, and then, with $x=0$, we shall obtain $\varphi\psi=\psi\varphi$. Then

$$\varphi^2x+u+\psi c+\psi x=\varphi x+u+\psi^2x+\psi c,$$

or

$$-u-\varphi x+\varphi^2x+u=\psi^2x+\psi c-\psi x-\psi c,$$

i.e.

$$I_u\varphi(-x+\varphi x)=\psi(\psi x-I_c^{-1}x). \qquad \square$$

The theorem implies the following result immedately.

Corollary 1.5. *The following classes of linear isotopes coinside:*

a) left semimedial = right semimedial = semimedial = medial;

b) left distributive = right distributive = distributive = idempotent;

c) CH = TS;

d) left ***Bol*** *= LIP;*

e) right ***Bol*** *= RIP;*

f) ***Bol = Mufang*** *= IP;*

g) left alternative = right alternative = alternative = loops = groups;

h) distributive Steiner quasigroups = Steiner quasigroups.

The pairs $<\varphi,\psi>$, $<\tilde{\varphi},\tilde{\psi}>$ of the unitary substitutions are said to be *midle-isoequal* (*left-isoequal, right-isoequal*) in a group $(Q;+)$, if there exist elements $a,b\in Q$, such that the isotopes $(Q;\cdot)$ and $(Q;*)$, defined by the equalities $xy=\varphi x+a+\psi y$ and $x*y=\tilde{\varphi}x+a+\tilde{\psi}y$ (respectively $xy=a+\varphi x+\psi y$ and $x*y=b+\tilde{\varphi}x+\tilde{\psi}y$, $xy=\varphi x+\psi y+a$ and $x*y=\tilde{\varphi}x+\tilde{\psi}y+b$), are isomorphic.

We recall some results, obtained by F. Sokhatsky in [3], in the following statement.

Theorem 1.6. *The following assertions are true.*

a) An isomorphism of group isotopes implies an isomorphism of the corresponding groups.

b) There exists a bijection between the sets of all isotopes of isomorphic groups such that the corresponding isotopes are isomorphic.

c) For every element 0 of a group isotope $(Q;f)$ *there exists exactly one quadruple* $(+,\alpha_1,\alpha_2,c)$, *such that* $(Q;+)$ *is a group with a neutral element 0, and* α_1,α_2 *are unitary substitutions of the group* $(Q;+)$ *and*

$$f(x,y)=a+\alpha_1 x+\alpha_2 y, \qquad g(x,y)=b+\beta_1 x+\beta_2 y \tag{3}$$

hold (the right side of the equality is called "left canonical decomposition").

d) If (3) are canonical decompositions of $(Q;f)$ *and* $(Q;g)$ *respectively, then the group isotopes are isomorphic if and only if there exist* $c\in Q$, $\theta\in Aut(Q,+)$, *such that:*

$$\theta b=a+\alpha_1 c+\alpha_2 c-c,$$
$$\theta\beta_1 x=c-\alpha_2 c-\alpha_1 c+\alpha_1(\theta x+c)+\alpha_2 c-c,$$
$$\theta\beta_2 x=c-\alpha_2 c+\alpha_2(\theta x+c)-c.$$

e) If pairs $<\varphi,\psi>$ *and* $<\tilde{\varphi},\tilde{\psi}>$ *of unitary substitutions are left-isoequal in a group* $(Q;+)$, *then there exists a bijection between the set of all isotopes of the type* (φ,ψ,L_a^{-1}) *and the set of all isotopes of the type* $(\tilde{\varphi},\tilde{\psi},L_b^{-1})$ *of the group* $(Q;+)$ *such that the corresponding isotopes are isomorphic.*

f) Every subquasigroup of an isotope of a group is a right coset of the group by some of its subgroup.

Theorem 1.7. *Linear isotopes* (φ, ψ, a) *and* $(\tilde{\varphi}, \tilde{\psi}, b)$ *of a group* $(Q;+)$ *are isomorphic if and only if there exist such* $c \in Q$, $\theta \in Aut(Q,+)$, *that*

$$\theta b = \varphi c + \psi c + a - c, \quad \theta\tilde{\varphi} = \varphi\theta, \quad I_{\varphi c}\theta\tilde{\psi} = \psi\theta.$$

Proof. Let us denote the operations in the given isotopes by f and g respectively. Then

$$f(x,y) = a + I_a\varphi x + I_a\psi y, \qquad g(x,y) = b + I_b\tilde{\varphi}x + I_b\tilde{\psi}y.$$

By **Theorem 1.6** d) $(Q;f) \cong (Q;g)$ if and only if there exist $c \in Q, \theta \in Aut(Q;+)$ such that

$$\theta b = a + I_a\varphi c + I_a\psi c - c,$$
$$\theta I_b\tilde{\varphi} = I_{I_a\varphi c + I_a\psi c - c} I_a\varphi\theta,$$
$$\theta I_b\tilde{\psi} = I_{I_a\psi c - c} I_a\psi\theta.$$

Accounting that $\theta I_b = I_{\theta b}\theta$, we'll substitute the first equality into the second and the third ones:

$$\theta\tilde{\varphi} = \varphi\theta, \qquad I_{\varphi c}\theta\tilde{\psi} = \psi\theta.$$

It remains to simplify the first equality. □

Theorem 1.8. *If pairs* $<\varphi, \psi>$ *and* $<\tilde{\varphi}, \tilde{\psi}>$ *of unitary substitutions are right-isoequal in a group* $(Q;+)$, *then there exists a bijection between the set of all isotopes of the type* $(\varphi, \psi, R_a^{-1})$ *and the set of all isotopes of the type* $(\tilde{\varphi}, \tilde{\psi}, R_b^{-1})$ *of the group, such that the corresponding isotopes are isomorphic.*

Proof. Really, then there exist $a, b \in Q$, for which the isotopes $(Q;\cdot)$ and $(Q;\times)$, defined by the equalities

$$xy = \varphi x + \psi y + a, \qquad x \times y = \tilde{\varphi}x + \tilde{\psi}y + b,$$

are isomorphic. Then $(Q;\bullet)\cong(Q;\otimes)$, where

$$x\bullet y = yx = \varphi y+\psi x+a = a\oplus\psi x\oplus\varphi y,$$
$$x\otimes y = b\oplus\widetilde{\psi}x\oplus\widetilde{\varphi}y,$$

i.e. $<\psi,\varphi>$ and $<\widetilde{\psi},\widetilde{\varphi}>$ are left-isoequal in the group $(Q;\oplus)$. By **Theorem 1.6** e) there exists a bijection between the set of isotopes of the type $(Q;f)$ and $(Q;g)$, where

$$f(x,y)=c\oplus\psi x\oplus\varphi y,$$
$$g(x,y)=d\oplus\widetilde{\psi}x\oplus\widetilde{\varphi}y,$$

for which the respective isotopes are isomorphic.

Then the commutations of the corresponding isotopes are isomorphic, but these are the isotopes (φ,ψ,R_c^{-1}) and $(\widetilde{\varphi},\widetilde{\psi},R_d^{-1})$ of the group $(Q;+)$. □

To describe the algorithm given below we have to cite the following evident assertion.

Proposition 1.9. *If* $(H;+)$ *is a subgroup of the group* $(Q;+)$, *then* $H+a=H+b$ *if and only if* $(a-b)\in H$.

2. A description of isotopes

Let us describe all linear group isotopes up to the 15-th order up to isomorphism. The obtained isotopes will be classified accoding to the known classes of quasigroups; the full list of subquasigroups of every isotope will be given.

By **Theorem 1.6** a),b) the problem can be solved for every of 28 groups of the indicated order separately (up to isomorphism). Let a group $(Q;+)$ be given, i.e. an order, a neutral element, generators and a **Cayley table** are known. Let us apply the following algorithm to the group.

Algorithm. First, using generators, we construct a sequence of the formation of all other elements (besides the neutral element). We verify also whether or not the group is abelian; we construct a table of all inverse elements of the group. We find all subgroups (except the group itself) of the given group, looking through all proper subsets, whose number of elements is a factor of the order of the group. It is enough to verify the closure of the subset under the main operation only. Furthermore, using **Proposition 1.9** we find all right cosets of the group by all subgroups, other than the given group. We find all automorphisms of the group. For this purpose consider all injectional mappings from the generator set into the group and their extentions to endomorphisms, i.e. using the properties $\varphi 0=0$, $\varphi(x+y)=\varphi x+\varphi y$. If its kernel is trivial, then it is an automorphism.

Remember the actions of all automorphisms on all elements of the group, and also find the identity automorphism (it moves no one of the generators in contrast to the others).

If the group is abelian, we find the automorphism $I=-\varepsilon$ as well (it maps every generator to its inverse and only I does it). Furthermore, we construct **Cayley table** for automorphisms. If φ and ψ are automorphisms of the group, then the only automorphism $\varphi\psi$ acts on every of the generators t, as φ acts on ψt. Construct a table for inverse automorphisms by verifying, if a composition coinsides with ε.

Let us find the correspondence $c \to I_c$ *(it is enough to verify* the actions of the automorphisms on generators). We create a table A of the size $k \times k$, where k is the number of all automorphisms and fill with "+". Consider, in turn, all pairs of automorphisms. Let $<\varphi, \psi>$ be a next in turn pair. If the respective box in the table A contains a sign "-", we consider the next pair. Otherwise, we create a table B, filled with "+", of the size being equal to the order of the group. For all pairs $<\theta, c> \in Aut(Q;+) \times Q$ we do the following: if

$$(\theta^{-1}\varphi\theta \neq \varphi) \vee ((I_{\varphi c}\theta)^{-1}\psi\theta \neq \psi)$$

then we put "-" in the box of the table A, corresponding to the pair $<\theta^{-1}\varphi\theta, (I_{\varphi c}\theta)^{-1}\psi\theta>$; otherwise, for every element m of the group (when the corresponding box contains "+" in the table B), if $n = \theta^{-1}(\varphi c + \psi c + m - c)$ has a number, which is greater than the number of the element m, then in the table B we put "-" in the box corresponding to the element n. As the result, we get all triples $<\varphi, \psi, a>$, where a runs the set of all elements of the group having the sign "+" in the table B.

Having run all the table A we obtain, according to **Theorems 1.7, 1.8**, a list of all linear isotopes of the group $(Q;+)$. Using **Theorems 1.4, 1.6** f), we select in this list the isotopes from the known classes of quasigroups and find all subquasigroups of every isotope.

This algorithm was applied to all 28 groups up to the 15-th order using a personal computer. Linear isotopes of the first and the second orders are isomorphic to groups of the same order.

Thus, we consider linear isotopes of order greater than two. To account the results on such groups, we need some designations.

The elements of the group H be denoted as follows:

- "0",...,"$m-1$", if $H = Z_m$;
- "xy", where $<x, y>$ is a corresponding vector, if $H = Z_m \times Z_2$ or $H = Z_3 \times Z_3$;
- $4a + 2b + c$, where $<a, b, c>$ is a corresponding vector, if $H = Z_2 \times Z_2 \times Z_2$;
- "xy", if $H = D_m$ (diedr group) and a corresponding element is obtained after the application of y symmetries with respect to a fixed axis of the m-angle and then of x elementary turns in the fixed direction;
- "1","i","j","k","-1","$-i$","$-j$","$-k$" as usual, if $H = Q_8$;
- "xy", where φ is a corresponding element and $\varphi 1 = x$, $\varphi 2 = y$, if $H = A_4$ is an alternating group;
- "mn", where $a^m b^n$ is a corresponding element, if

$$H = G_{12} = \{a^m b^n \mid m = 0,1,2,3; n = 0,1,2; a^4 = b^3 = 1; ba = ab^2\}.$$

As sequences of the generators select the following: "1" in cyclic groups; "4","2","1" in $Z_2 \times Z_2 \times Z_2$; "$i$","$j$" in Q_8; "13","21" in A_4; "10","01" in the others. The automorphisms will be denoted by a sequence of images of the generators.

All right cosets by all subgroups (except the group itself) will be numbered. The writing "N: $a_1,\dots,a_k + b_1,\dots,b_l$" in the next paragraph means that the number N is denoted a subgroup H created by the generators $a_1,\dots,a_k$, and the numbers $N+1,\dots,N+l$ are the cosets $H + b_1,\dots,H + b_l$ respectively.

Group Z_3. 1: 0+1,2.

Group Z_4. 1: 0+1,2,3; 5: 2+1.

Group Z_5. 1: 0+1,...,4.

Group Z_6. 1: 0+1,...,5; 7: 3+1,2; 10: 2+1.

Group Z_7. 1: 0+1,...,6.

Group Z_8. 1: 0+1,...,7; 9: 4+1,2,3; 13: 2+1.

Group Z_9. 1: 0+1,...,8; 10: 3+1,2.

Group Z_{10}. 1: 0+1,...,9; 11: 5+1,...,4; 16: 2+1.

Group Z_{11}. 1: 0+1,...,10.

Group Z_{12}. 1: 0+1,...,11; 13: 6+1,...,5; 19: 4+1,2,3; 23: 3+1,2; 26: 2+1.

Group Z_{13}. 1: 0+1,...,12.

Group Z_{14}. 1: 0+1,...,13; 15: 7+1,...,6; 22: 2+1.

Group Z_{15}. 1: 0+1,...,14; 16: 5+1,...,4; 21: 3+1,2.

Group $Z_2 \times Z_2$. 1: 00+01,10,11; 5: 01+10; 7: 10+01; 9: 11+01.

Group $Z_4 \times Z_2$. 1: 00+01,10,...,31; 9: 01+10,20,30; 13: 20+01,10,11; 17: 21+01,10,11; 21: 01,20+10; 23: 10+01; 25: 11+01.

Group $Z_6 \times Z_2$. 1: 00+01,10,...,51; 13: 01+10,20,...,50; 19: 30+01,10,...,21; 25: 31+01,10,...,21; 31: 20+01,10,11; 35: 01,30+10,20; 38: 01,20+10; 40: 10+01; 42: 11+01.

Group $Z_3 \times Z_3$. 1: 00+01,02,...,22; 10: 01+10,20; 13: 10+01,02; 16: 11+01,02; 19: 12+01,02.

Group $Z_2 \times Z_2 \times Z_2$. 1: 0+1,...,7; 9: 1+2,4,6; 13: 2+1,4,5; 17: 3+1,4,5; 21: 4+1,2,3; 25: 5+1,2,3; 29: 6+1,2,3; 33: 7+1,2,3; 37: 1,2+4; 39: 1,4+2; 41: 1,6+2; 43: 2,4+1; 45: 2,5+1; 47: 3,4+1; 49: 3,5+1.

Group D_3. 1: 00+01,10,...,21; 7: 01+10,11; 10: 11+01,20; 13: 21+01,10; 16: 10+01;

Group D_4. 1: 00+01,10,...,31; 9: 01+10,11,20; 13: 11+01,20,21; 17: 20+01,10,11; 21: 21+01,10,30; 25: 31+01,10,11; 29: 01,20+10; 31: 10+01; 33: 11,20+01.

Group D_5. 1: 00+01,10,...,41; 11: 01+10,11,20,21; 16: 11+01,20,21,30; 21: 21+01,10,30,31; 26 31+01,10,11,40; 31: 41+01,10,11,20; 36: 10+01.

Group D_6. 1: 00+01,10,...,51; 13: 01+10,11,...,30; 19: 11+01,20,21,30,31; 25: 21+01,10,30,31,40; 31· 30: +01,10,...,21; 37: 31+01,10,11,40,41; 43: 41+01,10,11,20,50; 49: 51+01,10,...,21; 55: 20+01,10,11; 59: 01,30+10,11; 62: 11,30+01,20; 65: 21,30+01,10; 68: 01,20+10; 70: 10+01; 72: 11,20+01.

Group D_7. 1: 00+01,10,...,61; 15 01+10,11,...,31; 22: 11+01,20,21,...,40; 29: 21+01,10,30,31,40,41; 36: 31+01,10,11,40,41,50; 43: 41+01,10,11,20,50,51; 50: 51+01,10,...,21,60; 57: 61+01,10,...,30; 64: 10+01.

Group Q_8. 1: **1+i,j,...,-k**; 9: **-1+i,j,k**; 13: **i+j**; 15: **j+i**; 17: **k+i**.

Group A_4. 1: 12+13,14,...,43; 13: 21+13,14,31,32,34; 19: 34+13,14,...,24; 25: 43+13,14,...,24; 31: 13+21,23,24; 35: 23+13,14,41; 39: 24+13,14,31; 43: 32+13,14,21; 47: 21,34+13,14.

Group G_{12}. 1: 00+01,02,...,32; 13: 20+01,02,...,12; 19: 01+10,20,30; 23: 10+01,02; 26: 11+01,02; 29: 12+01,02; 32: 01,20+10.

Now we number the sequences of the numbers of cosets (regardless of the groups). The notations "N: $a_1+b_1\cdot c_1,\dots,a_k+b_k\cdot c_k$" in the next paragraph means that with the number N (this will be an integer or an integer with a letter) a sequence

$$a_1, a_1+c_1, \dots, a_1+b_1c_1, a_2, a_2+c_2, \dots, a_2+b_2c_2, \dots, a_k+b_kc_k$$

is denoted. Here, instead of "$a+b\cdot 1$","$a+1\cdot c$", "$a+0\cdot c$","$a+b\cdot c, d+e\cdot f$" (where $d=a+bc$) we write "$a+b$","$a\oplus c$, a","$a+b\cdot c+e\cdot f$" respectively. The empty sequence will be denoted with the number 1.

2a: 1; 2b: 11; 2c: 17; 2d: 18; 2e: 19; 2f: 20; 2g: 21; 2h: 22; 2i: 23; 2k: 33; 2l: 34; 2m: 35; 2n: 36; 2o: 37; 2p: 39; 2q: 45; 2r: 46; 2s: 47; 2t: 48; 2u: 49; 2v: 50; 2w: 64; 2x: 65; 3a(4c): 1,5; 3b: 1,9; 3c: 1⊕9; 3d: 1,13; 3e: 1,16; 3f: 1,19; 3g: 1,25; 3h: 1,36; 3i: 1,39; 3j: 1,45; 3k: 1,64; 3l: 3⊕8; 3m: 3,16; 3n: 3,36; 3o: 3,64; 3p: 4,17; 3q: 4,37; 3r: 4,65; 3s: 5,16; 3t: 5,36; 3u: 5,49; 3v: 5,64; 3w: 6⊕4; 3x: 6,37; 3y: 6,65; 3z: 7,36; 3A: 7,64; 3B: 8,37; 3C: 8,65; 3D: 9,36; 3E: 9,64; 3F: 10,37; 3G: 10,65; 3H: 11,48; 3I: 11,64; 3J: 12,47; 3K: 12,65; 3L: 13,64; 3M: 14,65; 3N: 20⊕7; 3O: 20,33; 3P: 22⊕5; 3Q: 22,33; 3R: 32,43; 3S: 33,43; 3T: 58,71; 4a: 1,2; 4b: 1,4; 4c(3a): 1,5; 4d: 1,6; 4e: 1,7; 4f: 1,8; 4g: 31+1; 4h: 33+1; 5a: 1+2; 5b: 1,4,7; 5c: 1,5,9; 5d: 1,6,8; 5e: 10+2; 5f: 16+2; 5g: 21+2; 5h: 35+2; 6a: 1,7⊕3; 6b: 1,7⊕9; 6c: 1,11⊕5; 6d: 1,11,36; 6e: 1,13,39; 6f: 1,15⊕7; 6g: 1,15,64; 6h: 1,16⊕3; 6i: 1,16⊕5; 6j: 1,31⊕4; 6k: 2⊕5,17; 6l: 2⊕9,37; 6m: 2,15,65; 6n: 2,31,48; 6o: 3,31,49; 7a: 1,9⊕4; 7b: 1,9⊕6; 7c: 1+2·8; 7d: 1,13⊕8; 7e: 1,13,45; 7f: 1,13,47; 7g: 1,17,31; 7h: 1,25,39; 7i: 1,25,45; 7j: 1+2·24; 7k: 1,33,45; 7l: 2+2·8; 7m: 2,18,32; 7n: 2,26,48; 7o: 3⊕8⊕7; 7p: 3,19,31; 7q: 4⊕8⊕5; 7r: 4⊕9,47; 7s: 4,20,32; 7t: 5,23,49; 7u: 6⊕4⊕8; 7v: 7⊕4⊕7; 7w: 7+2·12; 7x: 8+2·4; 7y: 8⊕4⊕5; 7z: 8+2·12; 7A: 8,12,49; 7B: 10,23,49; 7C: 12⊕6,47; 8a: 1,2,39; 8b: 1,6,45; 8c: 1,7,49; 8d: 1,8,45; 8e: 5,8,49; 8f: 6⊕5,48; 9a: 1+3; 9b: 1,2,7,8; 9c: 1,4,6,7; 9d: 31+3, 10a: 1,5,7,9; 10b: 33⊕6+1⊕3; 10c: 58,69⊕2+1; 11a: 1,5,9⊕7; 11b: 1,6,8,19;

12a: 1,2,9+1; 12b: 1,2,13+1; 12c: 1,4⊕9+1; 12d: 1,4,33⊕3; 12e: 1,5⊕8⊕2; 12f: 1,6,9+1; 12g: 3,4⊕7+1; 12h: 3,8⊕3+1; 12i: 7,8⊕3+1; 13a: 1,13⊕6,32; 13b: 1,31,55,70; 13c: 2,14⊕5,32; 13d: 2,32,56,71; 13e: 3,15⊕4,32; 13f: 3,33,57,70; 13g: 4,34,58,71; 13h: 5,17⊕3,33; 13i: 9,33,55,70; 13j: 11⊕6⊕5,33; 13k: 11,35,57,70; 14a: 1,12,31,47; 14b: 2,7,31,48; 15a: 1+4; 15b: 16+4; 16a: 1,13⊕6⊕4⊕3; 16b: 1,13⊕6⊕4⊕9; 16c: 1,19,31⊕4⊕5; 16d: 1,25⊕6⊕4⊕7; 16e: 1,31,55,65⊕5; 16f: 4,16⊕4⊕3,33; 16g: 5,35,55⊕6⊕9; 16h: 5,35,55,66⊕4; 16i: 6,36,56⊕4,71; 16j: 8,32,58+1,71; 16k: 10+2·6+1,33; 16l: 12,36,58⊕2,71; 17a: 1,9+3·4; 17b: 1,13+2·6,47; 17c: 1+2·12⊕8,45; 17d: 4+2·9⊕6,47; 17e: 9⊕9+1⊕9,47; 18a: 1,9⊕4+2·2; 18b: 1,13⊕8+2·2; 18c: 1,17,29+2·2; 18d: 1,25,39⊕6⊕4; 18e: 2⊕8+2·3⊕2; 18f: 3,19,30+1⊕3; 18g: 6⊕4+2·3⊕2; 18h: 6,18,29⊕3⊕2; 18i: 7⊕4⊕3+1⊕3; 18j: 7,19,30+1⊕3; 18k: 8,20,30⊕2+1; 19a: 1,2,13+1,39; 19b: 1,2,31+1⊕3; 19c: 1,4,33⊕3,49; 19d: 1,7,31+2·2; 19e: 1,8,31⊕3+1; 20a: 1,7+3·3; 20b: 1⊕9+3·3; 21: 1,2,5,6,39; 22: 2,4,6,17; 23: 1,13,31,43⊕4; 24a: 4⊕9,47+2; 24b: 9⊕9,47+2; 25a: 1+3·3+2; 25b: 1,5,9⊕7+2; 26a: 1+2,16+2; 26b: 1,4,7⊕9+2; 27a: 1,31,55,68+2·2; 27b: 3,33,57,69+1⊕3; 27c: 6,36,56,68⊕3⊕2; 27d: 9,33,55,68+2·2; 27e: 11,35,57,69+1⊕3; 28: 3,5,8⊕2,31,49; 29a: 1+3·2+3; 29b: 1,5,9,31⊕4+2; 29c: 1,6,8⊕8+3; 29d: 1+4·5+2; 30: 1+6; 31a: 1,9+3·4+2·2; 31b: 1+2·8⊕4⊕8+2·2; 31c: 1+2·12⊕8+2·6⊕4; 31d: 2,9⊕9⊕4⊕7⊕3⊕2; 32a: 1+5·2,36; 32b: 2+4·2+1,37; 33: 1,11+5·5; 34a: 1,13⊕6⊕4+3·3; 34b: 1,31,55⊕4+2·3⊕5; 34c: 9,33,55⊕5⊕3⊕4⊕3; 35a: 1,5⊕5⊕3,47+2; 35b: 6,7,9⊕9,47+2; 36: 1+7; 37a: 1,4,5,8⊕5+3; 37b: 1,6+6; 38a: 1,2,5,6⊕7+3,39; 38b: 1,2,7,8,31+4; 39a: 1,13+3·6⊕4⊕3+2·2; 39b: 1,13,31⊕6,55⊕4⊕9+2·2;

39c: 2,13,32⊕6,56⊕3⊕9⊕3⊕2; 40: 1+8; 41: 1,9+5·4+2·2;
42a: 2+2·4,32+2·2,56⊕5,71; 42b: 2+2·4,32+2·2,56,66⊕5;
42c: 4+2,16+2⊕2⊕3,33; 42d: 4+2·4,32+2·2,58⊕7⊕6;
42e: 10+2⊕4+2⊕4+1,33; 43a: 1+7·2,64; 43b: 2+6·2+1,65; 44: 1,15+7·7;
45: 1,13+3·6+4·4; 46: 2,3,8⊕3+1⊕6,47+2; 47a: 1+5·2+5; 47b: 1+5·3+5;
48: 1+10; 49a: 1+3·4+3·2⊕4+3; 49b: 1,5,9,25⊕3+1⊕2⊕4+2⊕5;
50: 1,2,5,6,9+1,31+1⊕3+2; 51: 1+2,8⊕3⊕2,31,43⊕4+2; 52a: 1+11;
52b: 1+8⊕7+2; 53: 1+12; 54: 2+2·4⊕3,32+3·2,56⊕3⊕9⊕3⊕2;
55: 1+7·2+7; 56: 1,9+7·4+6·2; 57: 1,13+7·6⊕4+3·3+2·2; 58: 1+11,31+6;
59: 1+3·4+6·2⊕3+1⊕2⊕4+3+2·2; 60: 1+20; 61: 1+22.

Now we give a full list of the obtained isotopes. Every linear isotope (φ,ψ,c) will be imagined as follows: a sign for φ, comma (","), a sign for ψ, comma, a sign for c, a sequence of the signs "i" and central dot ("·") ("i" on the k-th place means the truth of the k-th property from the given below in the paragraph (*) list, and "·" the property is false), a number of the sequence of proper subquasigroups of the isotope, fullstop (" "). If c is a neutral element of the group, then the sign for neutral element and the comma before it are omitted; if φ is the same as in the privious isotope, then the sign for φ and the comma after it are omitted; if both, then the signs for φ and for ψ coincide; the sign "/c" will be written instead of "φ,ψ,c". If all properties are false, only one central dot will be written. If the proper subquasigroups in the isotope are absent, a number of the sequence of subquasigroups (i.e. 1) will be omitted.

Note, that the numeration of the subquasigroup sequences are selected in such a way that the upper semilattices of subquasigroups (the order is inclusion) are isomorphic for different isotopes iff in the corresponding integers are the same.

Commutativity of an isotope one can verify using the equality $\varphi = \psi$. An isotope is: idempotent (a peak) iff there are (is) all (at least one) one-element subsets in the full list of subquasigroups; a monoquasigroup if there is no subquasigroups; left (right) symmetrical, iff the group is abelian and the equality $\psi = -\varepsilon$ (respectively $\varphi = -\varepsilon$) holds (it is enough to verify it on the generators only); primary, iff the equality $\varphi = \psi$ only in groups Z_3 and $Z_3 \times Z_3$; a right loop, if the equality $\varphi = \varepsilon$ is true. There is no point in giving mediality for cyclic $(Z_3 - Z_{15})$ and nonabelian $(D_3 - D_7, Q_8, A_4, G_{12})$ groups. The subsets of left and right F-quasigroups in the set of isotopes on abelian groups coincide with the subset of medial ones. Left loops in that set are defined by the equality $\psi = \varepsilon$.

(*) Hence, with the signs "i" and "·" we denote

- for isotopes of cyclic groups 1) LIP-quasigroups, 2) RIP-quasigroups and 3) elastic quasigroups;
- for isotopes of non-cyclic abelian groups 1) medial quasigroups, 2) LIP-quasigroups, 3) RIP-quasigroups and 4) elastic quasigroups;
- for isotopes of nonabelian groups 1) - 2) left and right F-quasigroups, 3) LIP-quasigroups, 4) RIP-quasigroups, 5) left loops and 6) elastic quasigroups.

Group Z_3. 1,1iii2a. 2ii·2a. 2,1ii·2a. 2iii5a. /1iii.

Group Z_4. 1,1iii3a. 3ii·3a. 3,1ii·3a. 3iii3a.

Group Z_5. 1,1iii2a. 2·i·2a. 3·i·2a. 4ii·2a. 2,1i··2a. 2··i2a. 3·2a. 4i·i15a. /1i··i. 3,1i··2a. 2·2a. 3··i15a. /1··i. 4i··2a. 4,1ii·2a. 2·ii15a. /1·i·. 3·i·2a. 4iii2a.

Group Z_6. 1,1iii6a. 5ii·6a. 5,1ii·6a. 5iii29a. /1iii2b.

Group Z_7. 1,1iii2a. 2·i·2a. 3·i·2a. 4·i·2a. 5·i·2a. 6ii·2a. 2,1i··2a. 2··i2a. 3·2a. 4·2a. 5·2a. 6i·i30a. /1i··. 3,1i··2a. 2·2a. 3··i2a. 4·2a. 5··i30a. /1··. 6i··2a. 4,1i··2a. 2·2a. 3·2a. 4··i30a. /1··i. 5·2a. 6i··2a. 5,1i··2a. 2·2a. 3··i30a. /1··. 4·2a. 5··i2a. 6i··2a. 6,1ii·2a. 2·ii30a. /1·i·. 3·i·2a. 4·i·2a. 5·i·2a. 6iii2a.

Group Z_8. 1,1iii7a. 3ii·7a. 5ii·7a. 7ii·7a. 3,1ii·7a. 3iii7a. 5ii·7a. 7ii·7a. 5,1ii·7a. 3ii·7a. 5iii7a. 7ii·7a. 7,1ii·7a. 3ii·7a. 5ii·7a. 7iii7a.

Group Z_9. 1,1iii3c. 2·i·3c. 4·i·3c. 5·i·3c. 7·i·3c. 8i··3c. 2,1i··3c. 2··i25a. /1··i. 4·3c. 5··i25a. /1··. 7·3c. 8i·i52a. /1i··. /3i·i5e. 4,1i··3c. 2·3c. 4··i3c. 5·3c. 7·3c. 8i··3c. 5,1i··3c. 2··i25a. /1··. 4·3c. 5··i52a. /1··i. /3··i5e. 7·3c. 8i·i25a. /1i··. 7,1i··3c. 2·3c. 4·3c. 5·3c. 7··i3c. 8i··3c. 8,1ii·3c. 2·ii52a. /1·i·. /3·ii5e. 4·i·3c. 5·ii25a. /1·i·. 7·i·3c. 8iii25a. /1iii.

Group Z_{10}. 1,1iii6c. 3·i·6c. 7·i·6c. 9ii·6c. 3,1i··6c. 3··i47a. /1··i2c. 7·6c. 9i··6c. 7,1i··6c. 3·6c. 7··i6c. 9i·i47a. /1i··2c. 9,1ii·6c. 3·i·6c. 7·ii47a. /1·i·2c. 9iii6c.

Group Z_{11}. 1,1iii2a. 2·i·2a. 3·i·2a. 4·i·2a. 5·i·2a. 6·i·2a. 7·i·2a. 8·i·2a. 9·i·2a. 10ii·2a. 2,1i··2a. 2··i2a. 3·2a. 4·2a. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10i·i48a. /1i··. 3,1i··2a. 2·2a. 3··i2a. 4·2a. 5·2a. 6·2a. 7·2a. 8·2a. 9··i48a. /1··. 10i··2a. 4,1i··2a. 2·2a. 3·2a. 4··i2a. 5·2a. 6·2a. 7·2a. 8··i48a. /1··. 9·2a. 10i··2a. 5,1i··2a. 2·2a. 3·2a. 4·2a. 5··i2a. 6·2a. 7··i48a. /1··. 8·2a. 9·2a. 10i··2a. 6,1i··2a. 2·2a. 3·2a. 4·2a. 5·2a. 6··i48a. /1··i. 7·2a. 8·2a. 9·2a. 10i··2a. 7,1i··2a. 2·2a. 3·2a. 4·2a. 5··i48a. /1··. 6·2a. 7··i2a. 8·2a. 9·2a. 10i··2a. 8,1i··2a. 2·2a. 3·2a. 4··i48a. /1··. 5·2a. 6·2a. 7·2a. 8··i2a. 9·2a. 10i··2a. 9,1i··2a. 2·2a. 3··i48a. /1··. 4·2a. 5·2a.

6·2a. 7·2a. 8·2a. 9··i2a. 10i··2a. 10,1ii·2a. 2·ii48a. /1·i·. 3·i·2a. 4·i·2a. 5·i·2a. 6·i·2a. 7·i·2a. 8·i·2a. 9·i·2a. 10iii2a.

Group Z_{12}. 1,1iii16a. 5ii·16a. 7ii·16a. 11ii·16a. 5,1ii·16a. 5iii49a. /1iii3P. 7ii·16a. 11ii·49a. /1ii·3N. 7,1ii·16a. 5ii·16a. 7iii16a. 11ii·16a. 11,1ii·16a. 5ii·49a. /1ii·3N. 7ii·16a. 11iii49a. /1iii3P.

Group Z_{13}. 1,1iii2a. 2·i·2a. 3·i·2a. 4·i·2a. 5·i·2a. 6·i·2a. 7·i·2a. 8·i·2a. 9·i·2a. 10·i·2a. 11·i·2a. 12ii·2a. 2,1i··2a. 2··i2a. 3·2a. 4·2a. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10·2a. 11·2a. 12i·i53a. /1i··. 3,1i··2a. 2·2a. 3··i2a. 4·2a. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10·2a. 11··i53a. /1··. 12i··2a. 4,1i··2a. 2·2a. 3·2a. 4··i2a. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10··i53a. /1··. 11·2a. 12i··2a. 5,1i··2a. 2·2a. 3·2a. 4·2a. 5··i2a. 6·2a. 7·2a. 8·2a. 9··i53a. /1··. 10·2a. 11·2a. 12i··2a. 6,1i··2a. 2·2a. 3·2a. 4·2a. 5·2a. 6··i2a. 7·2a. 8··i53a. /1··. 9·2a. 10·2a. 11·2a. 12i··2a. 7,1i··2a. 2·2a. 3·2a. 4·2a. 5·2a. 6·2a. 7··i53a. /1··i. 8·2a. 9·2a. 10·2a. 11·2a. 12i··2a. 8,1i··2a. 2·2a. 3·2a. 4·2a. 5·2a. 6··i53a. /1··. 7·2a. 8··i2a. 9·2a. 10·2a. 11·2a. 12i··2a. 9,1i··2a. 2·2a. 3·2a. 4·2a. 5··i53a. /1··. 6·2a. 7·2a. 8·2a. 9··i2a. 10·2a. 11·2a. 12i··2a. 10,1i··2a. 2·2a. 3·2a. 4··i53a. /1··. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10··i2a. 11·2a. 12i··2a. 11,1i··2a. 2·2a. 3··i53a. /1··. 4·2a. 5·2a. 6·2a. 7·2a. 8·2a. 9·2a. 10·2a. 11··i2a. 12i··2a. 12,1ii·2a. 2·ii53a. /1·i·. 3·i·2a. 4·i·2a. 5·i·2a. 6·i·2a. 7·i·2a. 8·i·2a. 9·i·2a. 10·i·2a. 11·i·2a. 12iii2a.

Group Z_{14}. 1,1iii6f. 3·i·6f. 5·i·6f. 9·i·6f. 11·i·6f. 13ii·6f. 3,1i··6f. 3··i6f. 5··i55a. /1·2i. 9·6f. 11·6f. 13i··6f. 5,1i··6f. 3··i55a. /1·2i. 5··i6f. 9·6f. 11·6f. 13i··6f. 9,1i··6f. 3·6f. 5·6f. 9··i6f. 11·6f. 13i·i55a. /1i··2i. 11,1i··6f. 3·6f. 5·6f. 9·6f. 11··i55a. /1··i2i. 13i··6f. 13,1ii·6f. 3·i·6f. 5·i·6f. 9·ii55a. /1·i·2i. 11·i·6f. 13iii6f.

Group Z_{15}. 1,1iii6i. 2·i·6i. 4ii·6i. 7·i·6i. 8·i·6i. 11ii·6i. 13·i·6i. 14ii·6i. 2,1i··6i. 2··i29d. /1··i2e. 4i··47b. /1i··2h. 7·6i. 8·29d. /1·2c. 11i··29d. /1i··2d.

13·6i. 14i·i61a. /1i··. /3i··5g. /5i·i15b. 4,1ii·6i. 2·i·47b. /1·i·2h. 4iii6i. 7·ii47b. /1·i·2i. 8·i·6i. 11ii·6i. 13·i·6i. 14ii·6i. 7,1i··6i. 2·6i. 4i·i47b. /1i··2i. 7··i6i. 8·6i. 11i··6i. 13·6i. 14i··47b. /1i··2h. 8,1i··6i. 2·29d. /1·2c. 4i··6i. 7·6i. 8··i61a. /1··i. /3··i5g. /5··i15b. 11i··29d. /1i··2e. 13·47b. /1·2h. 14i··29d. /1i··2f. 11,1ii·6i. 2·i·29d. /1·i·2d. 4ii·6i. 7·i·6i. 8·i·29d. /1·i·2e. 11iii29d. /1iii2f. 13·i·6i. 14ii·29d. /1ii·2c. 13,1i··6i. 2·6i. 4i··6i. 7·6I. 8·47b. /1·2h. 11i··6i. 13··i47b. /1··i2i. 14i··6i. 14,1ii·6i. 2·ii61a. /1·i·. /3·i·5g. /5·ii15b. 4ii·6i. 7·i·47b. /1·i·2h. 8·i·29d. /1·i·2f. 11ii·29d. /1ii·2c. 13·i·6i. 14iii29d. /1iii2d.

Group $Z_2 \times Z_2$. 0110,0110iiii3b. 0111··i·4a. /01··i··1001iii·3b. 1011·ii·4a. /10·ii·. 0111,0110·i··4a. /01·i··. 0111i··i2a. 1001ii··2a. 1110i··i9a. /01i···. 1001,0110iii·3b. 0111i·i·2a. 1001iiii10a.

Group $Z_4 \times Z_2$. 1001,1001iiii31a. 1021·ii·18b. 1101iii·17a. 1121i·i·7d. 3001iii·31a. 1021,1001iii·18b. 1021iiii18b. 1101·ii·7d. 1121··i·7d. 3001iii·18b. 3021iii·18b. 1101,1001iii·17a. 1021·ii·7d. 1101iiii17a. 1121··i·7d. 3001iii·17a. 3101iii·17a. 1121,1001ii··7d. 1021·i··7d. 1101·i··7d. 1121i··i7d. 3001ii··7d. 3121i···7d. 3001,1001iii·31a. 1021iii·18b. 1101iii·17a. 1121i·i·7d. 3001iiii31a.

Group $Z_6 \times Z_2$. 1001,1001iiii39a. 1031iii·16c. 1130i·i·6j. 2130iii·16d. 2131i·i·6j. 5001iii·39a. 1031,1001iii·16c. 1031iiii16c. 1101·ii·19e. /01·ii·2m. 1130··i·19b. /01··i·2m. 2130·ii·19b. /10·ii·2n. 2131··i·19e. /10··i·2n. 5001iii·16c. 5031iii·16c. 1130,1001ii··6j. 1031·i··19b. /01·i··2m. 1130i··i6j. 2130·i··19d. /10·i··2n. 2131i···38b. /01i···2m. 4131i··i38b. /01i···2m. 5001ii··6j. 5130i···6j. 2130,1001iii·16d. 1031·ii·19b. /10·ii·2n. 1130··i·19d. /10··i·2n. 2130iiii49b. /10iiii3S. 2131··i·50a. /01··i·5h. /10··i·4h. /11··i·. 4130iii·16d. 5001iii·49b. /10iii·3R. 5031·ii·50a. /10·ii·. /20·ii·4g. /30·ii·5h. 2131,1001ii··6j. 1031·i··19e. /10·i··2n. 1130i···38b. /01i···2m. 2130·i··50a.

/01·i··5h. /10·i··4h. /11·i··. 2131i··i29b. /10i··i2k. 4131i···6j. 5001ii··29b. /10ii··2l. 5130i··i58a. /01i···5h. /10i···. /20i··i9d. 5001,1001iii·39a. 1031iii·16c. 1130i·i·6j. 2130iii·49b. /10iii·3R. 2131i·i·29b. /10i·i·2l. 5001iiii59a. /10iiii10b.

Group $Z_3 \times Z_3$. 0110,0110iiii29c. /01iiii2g. 0111··i·2a. 0112··i·2a. 0120··i·2a. 0121··i·26a. /01··i·. 0122··i·3f. 0211··i·11b. /01··i·2f. 0212··i·3e. 0220iii·6h. 0221··i·5a. /01··I·. 0222··i·2a. 1001iii·6h. 1002·ii·2a. 1011··i·2a. 1012·ii·3e. 1021··i·5a. /10··i·. 1022·ii·11b. /10·ii·2g. 1102·ii·3f. 1112··i·2a. 1121··i·5a. /10··i·. 1202·ii·26b. /01·ii·. 1222··i·5b. /10··i·. 2002iii·29c. /01iii·2f. 2012··i·2a. 2022··i·2a. 2122··i·2a. 0111,0110·i··2a. 0111i··i2a. 0112·5d. /01·. 0120·5a. /01·. 0121·2a. 0122·2a. 0220·i··5a. /01·I··. 0221·2a. 0222i···2a. 1001ii··2a. 1012·i··2a. 1021·5d. /10·. 1112i···2a. 1120·5a. /10·. 1220i··i40a. /01i···. 2002ii··2a. 2110i···2a. 2221i···2a. 0112,0110·i··2a. 0111·5d. /01·. 0112i··i2a. 0120·2a. 0121·2a. 0122·5a. /01·. 0220·i··2a. 0221i···2a. 0222·5a. /01·. 1001ii··2a. 1011·2a. 1022·i··5a. /10·i··. 1110i···2a. 1120·5c. /10·. 1222i··i40a. /01i···. 2002ii··2a. 2111i···2a. 2220i···2a. 0120,0110·i··2a. 0111·5a. /01·. 0112·2a. 0120i··i2a. 0121·2a. 0122·5c. /01·. 0210i···2a. 0211·5a. /01·. 0212·2a. 1001ii··2a. 1012·i··5d. /10·I··. 1022·i··2a. 1112·5c. /10·. 1121i···2a. 1211i··i40a. /01i···. 2002ii··2a. 2122i···2a. 2212i···2a. 0121,0110·i··26a. /01·I··. 0111·2a. 0112·2a. 0120·2a. 0121i··i25b. /01i··i. 0122·2a. 0210·5a. /01·. 0211·2a. 0212i···3e. 0220·i··11a. /01·i··2c. 1001ii··3e. 1002·i··2a. 1011·5d. /10·. 1022·i··2a. 1110·5a. /10·. 1120·2a. 1210i··i52b. /01I···. /11i··i5f. 1211·5b. /10·. 2002ii·i25b. /01ii··. 2120i···3e. 0122,0110·i··3f. 0111·2a. 0112·5a. /01·. 0120·5c. /01·. 0121·2a. 0122i··i3f. 0210·2a. 0211i···3f. 0212·5a. /01·. 0220·i··3f. 1001ii··3f. 1002·i··2a. 1012·i··5a. /10·i··. 1021·2a. 1110·5c. /10·. 1120i···3f. 1121·2a. 1210·5b. /10·. 2002ii··3f. 2210i···3f. 1001,0110iii·6h.

0111i·i·2a. 0112i·i·2a. 0120i·i·2a. 0121i·i·3e. 0122i·i·3f. 1001iiii20b. 2002iii·20b. 2002,0110iii·29c. /01iii·2f. 0111i·i·2a. 0112i·i·2a. 0120i·i·2a. 0121i·ii25b. /01i·i·. 0122i·i·3f. 1001iii·20b. 2002iiii60a. /01iiii5e.

Group $Z_2 \times Z_2 \times Z_2$. 124,124iiii31c. 125··i·19a. /1··i·2p. 126··i·7e. 134··i·7h. 136··i·4b. /1··I·. 137··i·2a. 146··i·4e. /1··i·. 147··i·8c. /1··i·2v. 156··i·3j. 157··i·2a. 165··i·8a. /1··i·2p. 174iii·18d. 236··i·12d. /1··I·. 237··i·3g. 241·ii·19c. /1·ii·2v. 243··i·2a. 245··i·4a. /1··I·. 247·ii·7j. 256··i·2a. 263··i·12d. /2··I·. 265··i·4a. /2··I·. 273··i·4e. /2··I·. 276··i·4e. /1··I·. 326iii·17c. 351·ii·7k. 376i·i·7i. 421iii·31c. 125,124·i··19a. /1·i··2p. 125i··i6e. 126·12c. /1·. 127·3d. 134·8a. /1·2p. 135·3I. 136·2a. 137·4b. /1·. 142·4e. /1·. 143·4e. /1·. 146·4e. /1·. 147·4e. /1·. 152·2a. 153·2a. 156·2a. 157·2a. 162·4b. /1·. 163·2a. 164·8a. /1·2p. 172·2a. 173·4b. /1·. 174·i··8a. /1·i··2p. 214·4a. /2·. 217·2a. 234·4a. /1·. 237·4b. /1·. 241·i··2a. 243·4d. /1·. 247·i··4f. /1·i··. 251·4b. /1·. 253·4d. /1·. 254·4a. /1·. 261·2a. 263·4d. /2·. 267·2a. 271·4e. /1·. 273·9c. /1·. /2·. /3·. 274·9b. /1·. /2·. /3·. 314·4a. /2·. 316·2a. 324·12b. /1·. 326·i··12c. /1·i··. 346·4f. /1·. 354·4a. /1·. 364·9b. /1·. /2·. /3·. 376·2a. 413·4e. /2·. 421ii··6e. 423·i··3d. 431·i··3i. 453·2a. 463·4f. /2·. 516·3a. /4·. 524i··i38a. /1i···2p. 526·37a. /1·. /4·. /5·. 534·21a. /1·2p. /4·2p. /5·2p. 546·3a. /1·. 576·3a. /1·. 126,124·i··7e. 125·12c. /1·. 126i··i7e. 134·4b. /1·. 135·2a. 136·2a. 137·4a. /1·. 142·4e. /2·. 143·9c. /1·. /2·. /3·. 147·9b. /1·. /2·. /3·. 152·4b. /2·. 153·8b. /2·2q. 156·8d. /2·2q. 157·4a. /2·. 163·4d. /1·. 164·2a. 165·4f. /1·. 172·2a. 174·i··8d. /1·i··2r. 175·2a. 234·4b. /1·. 235·2a. 241·i··2a. 243·4b. /1·. 245·2a. 247·i··4a. /1·i··. 253·2a. 254·2a. 261·4b. /2·. 263·2a. 265·2a. 267·4a. /2·. 271·4e. /2·. 274·4e. /1·. 324i···7e. 325·12c. /1·. 341·4d. /1·. 345·4f. /1·. 354·8d. /1·2r. 361·9c. /1·. /2·. /3·. 367·9b. /1·. /2·. /3·. 421ii··7e. 423·i··7e. 425·i··12c. /4·i··. 523·12e. /1·. 623ii··7e. 136,124·i··4b. /1·i··. 125·2a. 126·2a.

127·4a. /1·. 136i··i2a. 137·4a. /1·. 142·4b. /2·. 143·4d. /2·. 147·4a. /2·. 152·4e. /2·. 153·9c. /1·. /2·. /3·. 157·9b. /1·. /2·. /3·. 162·2a. 164·4f. /1·. 165·2a. 173·4d. /1·. 174·i··2a. 175·4f. /1·. 243·2a. 247·i··4a. /1·i··. 254·4f. /1·. 261·4e. /2·. 267·9b. /1·. /2·. /3·. 273i···2a. 345i···2a. 357·4a. /1·. 421ii··2a. 517i··i36a. /1i···. 652i···2a. 764i···2a. 137,124·i··2a. 125·4b. /1·. 126·4a. /1·. 127·2a. 136·4a. /1·. 137i··i2a. 142·4d. /2·. 143·4b. /2·. 146·4a. /2·. 152·9c. /1·. /2·. /3·. 153·4e. /2·. 156·9b. /1·. /2·. /3·. 163·2a. 164·2a. 165·4f. /1·. 172·4d. /1·. 174·i··4f. /1·I··. 175·2a. 241·i··4d. /1·i··. 245·2a. 253i···2a. 256·4a. /1·. 265·4e. /1·. 276·4a. /2·. 364i···2a. 376·9b. /1·. /2·. /3·. 421ii··2a. 516i··i36a. /1I···. 672i···2a. 745i···2a. 421,124iii·31c. 125i·i·6e. 126i·i·7e. 136i·i·2a. 137i·i·2a. 421iiii56a.

Group D_3. 1001,1001iiiiii20a. /01i·ii··6k. 10i··i··3s. 1011,1001·ii·i·3e. /01··i···3p. /10·3s. /20·3m. 2001,1001·iiii·6b. /01··ii··22a. /10···i··3m. /11··ii··2c.

Group D_4. 1001,1001iiiiii41a. /01iiii··31d. /10iiii··18j. 1011i··i··7g. /01i·ii··7z. 1011,1001·ii·i·7g. /01··i···7s. /10·ii···7w. 1011·7g. /01··i···7m. /10·7w. 1021,1001iiiii·18c. /01iiii··18h. /10iiii·i18j. 1011i··i··7g. /01i·ii··7s. 3001,1001iiiii·31b. /01iiii·i31d. /10iiii··18f. /11iiii··18k. 1011···i··7g. /01··ii··7s. 3011,1001·iiii·7g. /01··ii··7z. /10·iii··7p. 1011···i··7g. /01··ii··7m. /11··ii··7z.

Group D_5. 1001,1001iiiiii33a. /01i·ii··6l. /10i··i··3D. 2001i··i··6d. /01i··i··6l. 1011,1001·ii·i·3h. /01··i···3q. /10·3D. /20·3z. /30·3t. /40·3n. 2001·3h. /01·3B. 2001,1001·ii·i·6d. /01··i···32b. /10·3t. /11··i···2o. 2001·6d. /01·6l. /10·3z. /11·3q. 3001,1001·ii·i·6d. /01··i···6l. /10·3z. /11··i···3F. 2001·6d. /01·32b. /10·3n. /11·2o. 4001,1001·iiii·6d. /01··ii··6l. /10···i··3n. /11··ii··3x. 2001···i··32a. /01···i··6l. /10···i··2n. /11···i··3F.

Group D_6. 1001,1001iiiii57a. /01i·ii··39c. /10i··i··27e. 1011i··i··13b. /01i·ii··16l. /20iiii··34c. 1011,1001·ii·i·13b. /01··i···13g. /10·13k. /20·13i. 1011·13b. /01··i···13d. /10·13k. /20·ii···13i. 1021,1001·ii·i·27a. /01··i···27c. /10·27e. /20·27d. 1011·13b. /01··i···13g. /10·13k. /20·ii···13i. 1031,1001iiiii·34b. /01i·ii··16j. /10i··i··13k. 1011i··i··13b. /01i·ii··16i. /20iiii·i34c. 5001,1001·iiii·39b. /01··ii··54a. /10···i··27b. /11··ii··10c. 1011···i··13b. /01··ii··3T. /11··ii··42a. /20·iii··16g. 5011,1001·iiii·16e. /01··ii··3T. /10···i··13f. /21··ii··42d. 1011···i··13b. /01··ii··42b. /11··ii··3T. /20·iii··16h.

Group D_7. 1001,1001iiiii44a. /01i·ii··6m. /10i··i··3L. 2001i··i··6g. /01i··i··6m. 3001i··i··6g. /01i··i··6m. 1011,1001·ii·i·3k. /01··i···3r. /10·3L. /20·3I. /30·3E. /40·3A. /50·3v. /60·3o. 2001·3k. /01·3G. 3001·3k. /01·3K. 2001,1001·ii·i·6g. /01··i···43b. /10·3A. /11··i···2x. 2001·6g. /01·6m. /10·3v. /11·3r. 3001·6g. /01·6m. /10·3I. /11·3G. 3001,1001·ii·i·6g. /01··i···6m. /10·3v. /11··i···3M. 2001·6g. /01·43b. /10·3I. /11·2x. 3001·6g. /01·6m. /10·3E. /11·3r. 4001,1001·ii·i·6g. /01··i···6m. /10·3I. /11··i···3C. 2001·6g. /01·6m. /10·3E. /11·3M. 3001·6g. /01·43b. /10·3o. /11·2x. 5001,1001·ii·i·6g. /01··i···6m. /10·3E. /11··i···3y. 2001·6g. /01·6m. /10·3o. /11·3C. 3001·43a. /01·6m. /10·2w. /11·3M. 6001,1001·iiii·6g. /01··ii··6m. /10···i··3o. /11··ii··3K. 2001···i··43a. /01···i··6m. /10···i··2w. /11···i··3y. 3001···i··6g. /01···i··6m. /10···i··3L. /11···i··3C.

Group Q_8. ij,ij··ii··7c. /i···i··7u. /k··ii··7y. ik···i··12a. /i···i··. ji·iiii·7c. /i··ii··7o. /k·iii··7q. jk···i··12a. /i··ii··12h. /j···i··. /k··ii··. ik,ij··i···12f. /i·. /j·12i. /k··i···. ik·3b. /i·3w. ji·ii·i·3b. /i··i···3l. kj·37b. /I·. i··j,ij··i···7c. /i·7l. /j·7v. ik·12f. /i·. ji·ii·i·7c. /i··i···7o. /k·ii···7y. jk·12f. /i··i···12g. /j·. /k··i···.

ik·12f. /i·. ji·ii·i·7c. /i··i···7o. /k·ii···7y. jk·12f. /i··ı···12g. /j·. /k··i···. ji,iji·ii··7c. /ii··i··7v. iki··i··3b. jiiiiiii18a. /iiiii··18g. j-i,ij··ii··7c. /i···i··7v. ik···i··3b. jiiiiii·18a. /iiiii··18e. /jiiii·i18i. jki··i··7b. /ii·ii··7x.

Group A_4. 1321,1321iiiiii45a. /13i··i··6o. /21i·ii··17d. 1421i·ii··23a. /23i··i··7B. 1334,1321·ii·i·6k. /13·28a. /14·6n. /21··i···3J. /23·3H. /24·2u. 1421··i···14a. /21··i···2s. /23·2u. /24·8f. 1421,1321·iiii·23a. /13···i··14b. /21··ii··7r. /23···i··2u. /34··ii··7C. 1421··ii··51a. /13··ii··2j. /21··ii··24a. /23···I··. /34···i··35b. 2321,1321·ii·i·7f. /13·2t. /14·8e. /21··i···7r. /34··i···7C. 1421··i···35a. /13··i···. /23·. /34·24b. /43·46a. 2421,1321·iiii·17b. /13···i··3u. /21··ii··17d. /34··ii··17e. 1421··ii··7f. /14··ii··7t. /23···i··7A. /24···i··7n.

Group G_{12}. 1001,1001iiiiii34a. /01i· i··13e. /10i·ii··16k. 3001iiii··34a. /01i··i··13e. /10i·ii··16f. 1002,1001·iiii·16b. /01···i··13c. /10··ii··42e. /11··ii··3Q. 3001·iii··16b. /01···i··13c. /10··ii··42c. /11··ii··3O. 1101,1001·ii·i·13a. /01·13e. /02·13c. /10··i···13j. 3001·ii···13a. /01·13e. /02·13c. /10··i···13h. 3001,1001iiiii·34a. /01i··i··13e. /10i·ii··16f. 3001iiii·i34a. /01i··i··13e. /10i·ii··16k. 3002,1001·iiii·16b. /01···i ·13c. /10··ii··42c. /11··ii··3O. 3001·iii··16b. /01···i··13c. /10··ii··42e. /11··ii··3Q. 3101,1001·ii·i·13a. /01·13e. /02·13c. /10··i···13h. 001·ii···13a. /01·13e. /02·13c. /10··i···13j.

Hence, we can sum up the number characteristics (up to isomorphism) of group isotopes up to the 15-th order in the following table. In the table it is denoted with V_{4m} the direct product of the group Z_{2m} by the group Z_2, with Z_2^3 the direct product of the group V_4 by the group Z_2, and with Z_3^2 the direct product of the group Z_3 by itself.

order	2	3	4		5	6		7	8				
Group	Z_2	Z_3	Z_4	V_4	Z_5	Z_6	D_3	Z_7	Z_8	V_8	Z_2^3	D_4	Q_8
a number of linear isotopes	1	5	4	15	19	5	11	41	16	28	341	28	47

9		10		11	12					13	14		15
Z_9	Z_3^2	Z_{10}	D_5	Z_{11}	Z_{12}	V_{12}	D_6	A_4	G_{12}	Z_{13}	Z_{14}	D_7	Z_{15}
48	183	19	37	109	20	75	44	43	44	155	41	79	95

Thus, there exist exactly 1554 linear isotopes of the group up to the 15-th order inclusively. One can conclude that there exist exactly 61 upper semilattices of subquasigroups of the linear isotopes up to isomorphism.

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